1. Three sets A, B and C are given. f is a function which is one-to-one onto and is defined from A to B. Also g is a function defined from C to A. If h = fog, then which of the following is the representation for g ?

A) $h^{-1}of$ B) hof C) $f^{-1}oh$ D) foh^{-1}

2. A representation of the rational number 5/6 as ternary expansion is given by

A) 0.2222... B) 0.2111... C) 0.2211... D) 0.2221...

3. Given $\{a_n\}$ is a sequence of real numbers. Now consider the following statements :

a) If $\{a_n\}$ converges to a limit a, then any subsequence of $\{a_n\}$ is also convergent to a.

b) $\{a_n\}$ may not converge to a limit, but still we can find such a_n's so that $\{|a_n|\}$ converges to a limit.

Now state which of the following is correct ?

A) Only (a) is true B) Only (b) is true

C) Neither (a) nor (b) is true D) Both (a) and (b) are true

4. Suppose Δ is defined as an operation between two sets A and B by $A \Delta B = (A \cup B) - (A \cap B)$. Further let X_1 and X_2 be two non empty sets such that E_1, E_2 are subsets of X_1 and F_1, F_2 are subsets of X_2. Then which of the following is correct ?

A) $(E_1 \Delta E_2) \times (F_1 \Delta F_2) = [(E_1 - E_2) \times (F_1 - F_2)] \cup [(E_1 - E_2) \times (F_2 - F_1)] \cup$
$[(E_2 - E_1) \times (F_1 - F_2)] \cup [(E_2 - E_1) \times (F_2 - F_1)]$

B) $(E_1 \Delta E_2) \times (F_1 \Delta F_2) = [E_1 \times (F_1 - F_2)] \cup [E_2 \times (F_2 - F_1)]$

C) $(E_1 \Delta E_2) \times (F_1 \Delta F_2) = [(E_1 - E_2) \times F_1] \cup [(E_2 - E_1) \times F_2]$

D) $(E_1 \Delta E_2) \times (F_1 \Delta F_2) = [(E_1 \cup E_2) \times (F_1 \cup F_2)] - [(E_1 \cap E_2] \times (F_1 \cap F_2)]$

5. Suppose $\{E_i\}$ and $\{F_j\}$ are two sequences of open sets. Then which of the following is essentially an open set ?

A) $\left(\bigcup_{i=1}^{\infty} E_i\right) \cup \left(\bigcup_{j=1}^{\infty} F_j\right)$ B) $\left(\bigcap_{i=1}^{\infty} E_i\right) \cup \left(\bigcap_{j=1}^{\infty} F_j\right)$

C) $\left(\bigcup_{i=1}^{\infty} E_i\right) \cap \left(\bigcap_{j=1}^{\infty} F_j\right)$ D) $\left(\bigcap_{i=1}^{\infty} E_i\right) \cap \left(\bigcap_{j=1}^{\infty} F_j\right)$

6. Consider the following statements on a metric space (X, ρ)

   a) Every finite subset of X is closed

   b) If A and B are open subsets of X, then $A \times B$ is an open subset of $X \times X$

   c) Every subspace of a complete matrix space is complete.

   State which of the following is correct ?

   A) All (a), (b) and (c) are true
   B) Only (a) is true
   C) Only (b) is true
   D) Only (a) and (b) are true

7. Let $I = (0, 1)$, $I^{(2)} = \{(x, y) \mid 0 < x < 1, 0 < y < 1\}$ and let C be the class of all disjoint open intervals contained in I. Suppose N is the set of all positive integers. Now state which of the following is correct ?

   A) C is equivalent (~) to a set of finite elements
   B) $C \sim I$
   C) $C \sim I^{(2)}$
   D) $C \sim N$

8. Given a finite dimensional vector space V. Let W be a subspace of V. Define further a space $V' = \{v + W = \{v + w \mid w \in W\}, v \in V\}$. Now consider the following Assertion (A) and Reason (R).

   A : If v + W = W, then $v \in W$

   R : V′ is the quotient space of V by W.

   Which of the following is then correct ?

   A) A is true but R is not
   B) A is not true but R is correct
   C) Both A and R are true, but R is not the correct reason for A
   D) Both A and R are not true

9. Consider the vectors (2, 0, – 4), (1, 1, –1) and (1, 0, 1) and the following Assertion (A) and Reason (R) :

   A : The given vectors are linearly independent

   R : The matrix formed by the above vectors is non-singular

   Which of the following is then correct ?

   A) A is true but R is not
   B) R is true but A is not
   C) Both A and R are false
   D) Both A and R are true and R is the correct reason for A

10. Consider the following linear transformation :

$y_1 = x_1 + x_2 + x_3,\ \ y_2 = x_1 - x_2 + x_3,\ \ y_3 = 2x_1 + x_2 - x_3$

Given the following statements :

i) The above linear transformation is one-to-one

ii) The Jacobian of the transformation is equal to 6

iii) The rank of the linear transformation is equal to 3.

Now state which of the following is correct ?

A) Only (i) and (iii) are true
B) Only (ii) and (iii) are true
C) Only (i) and (ii) are true
D) (i), (ii) and (iii) are all true

11. Given the matrix $A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$ and the following two list of items

List – 1	**List – 2**
i) Characteristic roots	a) $A^3 + 3A^2 - 4\,I_{3\times 3}$
ii) Zero matrix of order 3	b) 2, 2, –1
iii) A^{-1}	c) $4A - 3A^3$
iv) A^4	d) $\frac{1}{4}(A^2 + 3A)$
	e) 1, –2, –2

Which of the following is then the correct match ?

A) (i) – (e), (ii) – (a), (iii) – (d), (iv) – (c)
B) (i) – (b), (ii) – (d), (iii) – (c), (iv) – (a)
C) (i) – (e), (ii) – (c), (iii) – (d), (iv) – (a)
D) (i) – (e), (ii) – (c), (iii) – (a), (iv) – (d)

12. Suppose $\{E_n\}$ is a nonincreasing sequence of subsets of X. Then which of the following is correct ?

A) $\lim_{n\to\infty} E_n = \bigcup_{n=1}^{\infty} E_n$
B) $\lim_{n\to\infty} E_n = \bigcap_{n=1}^{\infty} E_n$
C) $\lim_{n\to\infty} E_n = \phi$
D) $\lim_{n\to\infty} E_n = X$

13. A Stieltjes measure function F(x) is given by

$$F(x) = \begin{cases} 0, & \text{for } x < 0 \\ 1 - e^{-x} - xe^{-x}, & \text{for } x \geq 0 \end{cases}$$

Now consider the following statements on Lebesgue – Stieltjes measure μ_F (a, b] on the interval (a, b] $\subset$ R.

i) $\mu_F(-1, \log 2] = \dfrac{1 + \log 2}{2}$

ii) $\mu_F(\log 2, \log 3] = \dfrac{1 + \log(8/9)}{6}$.

Now state which of the following is correct ?

A) Only (i) is true

B) Both (i) and (ii) are true

C) Only (ii) is true

D) Neither (i) nor (ii) is true

14. Let $\{f_n\}$ be a sequence of measurable functions on the Lebesgue measure space $(R, \mathscr{B}, \lambda)$ defined by

$$f_n(x) = \begin{cases} 1, & \text{for } x \in [n, n+1] \\ 0, & \text{otherwise} \end{cases}$$

for n = 1, 2, ... Now consider the following statements :

i) $f_n \to 0$ pointwise

ii) $f_n \to 0$ a.e.

iii) $f_n \to 0$ in measure

Which of the following is then correct ?

A) Only (i) and (ii) are true

B) Only (i) and (iii) are true

C) Only (ii) and (iii) are true

D) (i), (ii) and (iii) are all true

15. The value of $\int_{-1}^{3} \frac{|x|}{x} dx$ is equal to

A) 4

B) 2

C) 3

D) The integral does not exist

16. Given events A, B and C such that $P(A \cap B) = \frac{1}{2} P(B \cap C)$ and $P(A \cap C) = 0$. Then state which of the following is correct ?

A) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B \cap C)$

B) $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

C) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - 3P(A \cap B)$

D) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - 3P(B \cap C)$

17. A random experiment has n outcomes say $a_1, a_2, ..., a_n$. It is known that the outcome a_{j+1} is thrice as likely as outcome a_j for $j = 1, 2, ..., n - 1$. Then the probability of the elementary event $E_j = \{a_j\}$ is equal to

A) $\frac{1}{3^j}$

B) $\frac{2 \times 3^{j-1}}{3^n - 1}$

C) $\frac{3^{j-1}}{3^n - 1}$

D) $\frac{2 \times j}{n(n+1)}$

18. A box contains 'a' red and 'b' white balls. Balls are drawn from the box one after another without replacement. Suppose E_n is the event that a red ball is drawn for the first time on the n^{th} draw. Then $P(E_n)$ for $n \le \text{Min}(a, b)$ is equal to

A) $\frac{a}{a+b-n+1} \prod_{j=1}^{n-1} \left\{1 - \frac{a}{a+b-j+1}\right\}$

B) $\frac{a}{a+b} \left(1 - \frac{a}{a+b}\right)^{n-1}$

C) $\frac{b}{a+b} \left(1 - \frac{b}{a+b}\right)^{n-1}$

D) $\frac{b}{a+b-n+1} \prod_{j=1}^{n-1} \left\{1 - \frac{b}{a+b-j+1}\right\}$

19. Given n events $E_1, E_2, ..., E_n$ and consider the following statements :

i) $P\left(\bigcup_{i=1}^{n} E_i\right) \le \sum_{i=1}^{n} P(E_i)$

ii) $P\left(\bigcap_{i=1}^{n} E_i\right) \ge \sum_{i=1}^{n} P(E_i) - n + 1$

iii) $P\left(\bigcup_{i=1}^{n} E_i\right) \le \sum_{i=1}^{n} P(E_i) - \sum_{i} \sum_{j} P(E_i \cap E_j)$ (summation over $i < j$).

Which of the following is then true ?

A) (i) only

B) (ii) only

C) both (i) and (ii)

D) (i), (ii) and (iii)

20. Consider families with exactly three children. Let A be the event that the family has children of both sexes. Let B be the event that there is atmost one girl and let C be the event that there is atmost one boy. Consider now the following statements :

i) A and B are independent

ii) A and C are independent

iii) B and C are independent

Then which of the following is true ?

A) (i) and (ii) B) (i) and (iii) C) (ii) and (iii) D) (i), (ii) and (iii)

21. There are three identical urns. The first urn contains 5 white and 4 red balls while the second urn contains 3 white and 6 red balls and the third urn contains 2 white and 7 red balls. A person chooses an urn at random and draws a ball from it. The ball is white. What is the probability that the ball is drawn from the second urn ?

A) $\frac{5}{10}$ B) $\frac{3}{10}$ C) $\frac{1}{3}$ D) $\frac{2}{10}$

22. A die is rolled until a face with number 6 turns up. Let A_k be the event that the face of die with number 6 appears for the first time at k. Then $P(A_k)$ is equal to

A) $\frac{1}{6}\left(\frac{1}{6}\right)^{k-1}, k = 1, 2, \ldots$ B) $\frac{1}{6}\left(\frac{5}{6}\right)^{k-1}, k = 1, 2, \ldots$

C) $\frac{5}{6}\left(\frac{1}{6}\right)^{k-1}, k = 1, 2, \ldots$ D) $\frac{5}{6}\left(\frac{5}{6}\right)^{k-1}, k = 1, 2, \ldots$

23. Consider the following functions :

i) $G_1(x) = \left[\frac{10}{10 + x}\right]^3, 0 \leq x < \infty$

ii) $G_2(x) = e^{-x} + xe^{-x}, 0 \leq x < \infty$

iii) $G_3(x) = [1 + e^{-x}]^{-1}, -\infty < x < \infty$

Then which of the following is correct ?

A) Only (i) and (ii) are survival functions

B) Only (i) and (iii) are survival functions

C) Only (ii) and (iii) are survival functions

D) (i), (ii) and (iii) are all survival functions

24. If f(x) is the pdf and the corresponding cdf is F(x), then consider the following two list of items :

List – 1	List – 2
i) $6\,F(x)\,f(x) - 6\,[F(x)]^2\,f(x)$	a) cdf of the median in a sample of size 3
ii) $f(x)/F(y),\ -\infty < x < y$	b) pdf of the median of a sample of size 3
iii) $3\,[F(x)]^2 - 2[F(x)]^3$	c) right truncated distribution of f(x)
	d) conditional pdf of the largest order statistic $X_{3:3}$ given the smallest ordered observation $X_{1:3} = y$

Which of the following is then the correct match ?

A) (i) – (b), (ii) – (d), (iii) – (a)
B) (i) – (d), (ii) – (c), (iii) – (b)
C) (i) – (b), (ii) – (c), (iii) – (a)
D) (i) – (a), (ii) – (c), (iii) – (b)

25. If F(x, y) is the cdf of a bivariate random vector (X, Y), then $P(x_1 < X \leq x_2, y_1 < Y \leq y_2)$ is equal to

A) $F(x_2, y_2) - F(x_1, y_1)$
B) $F(x_2, y_2) - F(x_1, y_2) - F(x_2, y_1) + F(x_1, y_1)$
C) $F(x_2, y_2) - F(x_1, y_2) + F(x_2, y_1) - F(x_1, y_1)$
D) $F(x_2, y_2) + F(x_1, y_2) - F(x_2, y_1) - F(x_1, y_1)$

26. Which of the following is the pdf of a random variable with moment generating function $m(t) = \frac{1}{1-2t}$?

A) $f(x) = 2e^{-2x},\ x > 0$
B) $f(x) = \frac{1}{2} e^{-\frac{x}{2}},\ x > 0$
C) $f(x) = xe^{-\frac{x^2}{2}},\ x > 0$
D) $f(x) = 2x\,e^{-x^2},\ x > 0$

27. Which of the following is the pdf of a random variable whose characteristic function has the expression $\varphi(t) = e^{i\mu t - \sigma|t|}$?

A) $f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$
B) $f(x) = \frac{1}{\sigma} \frac{\exp[(x-\mu)/\sigma]}{\{1+\exp[(x-\mu)/\sigma]\}^2}$
C) $f(x) = \frac{1}{2\sigma} e^{-|[(x-\mu)/\sigma]|}$
D) $f(x) = \frac{1}{\pi\sigma} \frac{1}{1+[(x-\mu)/\sigma]^2}$

28. If $\mu_{(r)}$ is the r^{th} factorial moment and μ'_r is the r^{th} raw moment, then which of the following is correct ?

A) $\mu_{(3)} = \mu'_3 - 2\mu'_2 + \mu'_1$

B) $\mu_{(3)} = \mu'_3 - 3\mu'_2 + 3\mu'_1$

C) $\mu_{(3)} = \mu'_3 - 3\mu'_2 + 2\mu'_1$

D) $\mu_{(3)} = \mu'_3 + 3\mu'_2 + 2\mu'_1$

29. If X is a continuous type random variable which is symmetric about zero, then consider the following statements :

i) $E(X) = 0$

ii) Mode is zero

iii) Median is zero

Now state which of the following is always correct ?

A) (i) and (ii)

B) (i) and (iii)

C) only (iii)

D) all (i), (ii) and (iii)

30. Suppose $\{X_n\}$ and $\{Y_n\}$ are two sequences of random variables, X is another random variable and $c \neq 0$ is a constant. Now consider the following two lists such that items in List – 2 are the possible implications of the items in List – 1.

List – 1	List – 2
i) $X_n \xrightarrow{L} X, Y_n \xrightarrow{P} c \Rightarrow X_n + Y_n$	a) $\xrightarrow{L} c^{-1}X$
ii) $X_n \xrightarrow{L} X, Y_n \xrightarrow{P} c \Rightarrow X_n Y_n$	b) $\xrightarrow{L} X + c$
iii) $X_n \xrightarrow{L}, Y_n \xrightarrow{P} c \Rightarrow X_n / Y_n$	c) $\xrightarrow{P} cX$
	d) $\xrightarrow{P} X + c$
	e) $\xrightarrow{L} cX$

Which of the following is then the correct match ?

A) (i) – (d), (ii) – (c), (iii) – (a)

B) (i) – (d), (ii) – (e), (iii) – (a)

C) (i) – (b), (ii) – (e), (iii) – (a)

D) (i) – (b), (ii) – (c), (iii) – (a)

31. Suppose X is a binomial random variable with n = 4 and parameter p. Given P(X = 3) = 4P (X = 1). Then the value of p is

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{1}{4}$ D) $\frac{3}{4}$

32. The moment generating function of a random variable with the set of non-negative integers as support set is $M(t) = e^{4(e^t - 1)}$. Then P(X = 4) is equal to

A) $\left(\frac{1}{2}\right)^4$ B) e^{-4} C) $\frac{32}{3}e^{-4}$ D) $\frac{3}{32}e^{-4}$

33. Let $X_1, X_2, ..., X_n$ be a random sample of size n from a distribution with pdf,

$f(x) = \frac{1}{\pi\sigma} \frac{1}{1 + \frac{(x-\mu)^2}{\sigma^2}}$, $-\infty < x < \infty$. Then the distribution of $\bar{X} = (X_1 + ... + X_n)/n$ is given by the pdf

A) $g(x) = \frac{1}{\pi\sigma} \frac{1}{1 + \frac{(x-\mu)^2}{\sigma^2}}$

B) $g(x) = \frac{\sqrt{n}}{\pi\sigma} \frac{1}{1 + \frac{n(x-\mu)^2}{\sigma^2}}$

C) $g(x) = \frac{n}{\pi\sigma} \frac{1}{1 + \frac{(x-\mu)^2}{\sigma^2} \cdot n^2}$

D) $g(x) = \frac{1}{\pi\sigma} \frac{1}{\frac{(x-n\mu)^2}{\sigma^2} + 1}$

34. Suppose X is a non-negative continuous random variable with cdf F(x). Further if $\frac{1-F(x+y)}{1-F(y)} = 1 - F(x)$, for all x, y > 0, then the distribution of X is

A) Uniform over (0, 1)

B) F-distribution

C) Negative exponential

D) Gamma

35. Suppose X is a random variable with pdf f(x) given by $f(x) = x e^{-\frac{x^2}{2}}$, x > 0, then the distribution of $Y = 1 - e^{\frac{-x^2}{2}}$ is

A) Negative exponential

B) Uniform over (0, 1)

C) Weibull distribution

D) Half normal distribution

36. The random variable X has the pdf f(x, α) given by $f(x, \alpha) = \begin{cases} \alpha x^{\alpha-1} & , 0 < x < 1 \\ 0 & , \text{otherwise} \end{cases}$.

It is known that $P(X \leq 0.5) = 0.125$. Now consider the statements on the random variable X

i) Mean $= \frac{3}{4}$

ii) Median $= \frac{3}{8}$

iii) Variance $= \frac{3}{80}$.

Then state which of the following is correct ?

A) (i) and (iii)

B) (i) and (ii)

C) (ii) and (iii)

D) all (i), (ii) and (iii)

37. Let $X_{1:n}, X_{2:n}, \ldots, X_{n:n}$ be the order statistics of a random sample of size n drawn from a Bernoulli distribution with probability of success equal to π. Then $X_{r:n}$ is distributed again as Bernoulli with probability of success π_r given by

A) $\pi_r = \sum_{i=r}^{n} \binom{n}{i} \pi^i (1-\pi)^{n-i}$

B) $\pi_r = \sum_{i=n-r+1}^{n} \binom{n}{i} \pi^i (1-\pi)^{n-i}$

C) $\pi_r = \sum_{i=r}^{n} \binom{n}{i} (1-\pi)^i \pi^{n-i}$

D) $\pi_r = \sum_{i=n-r+1}^{n} \binom{n}{i} (1-\pi)^i \pi^{n-i}$

38. Let R be the range of a random sample of size n drawn from an absolutely continuous distribution with cdf F(x) and pdf f(x). Then which of the following represents the cdf G(R) of R ?

A) $G(R) = n\int_{-\infty}^{\infty} [F(x+R) - F(x)]^{n-1} dF(x)$

B) $G(R) = \int_{-\infty}^{\infty} \{1 - [F(x+R)]^n - [1-F(x)]^n\} f(x+R)\, dF(x)$

C) $G(R) = n(n-1)\int_{-\infty}^{\infty} [F(u+R) - F(u)]^{n-2} f(u)\, f(u+R)\, du$

D) $G(R) = n(n-1)\int_{0}^{R} [F(x+R) - F(x)]^{n-1} dF(x)$

39. The difference between pre-treatment and post-treatment scores of five subjects in a trial are 1.5, 1.0, 2.5, 1.5 and 2.5. Then the value of the appropriate t-statistic for testing the effectiveness of the treatment is equal to

A) 4.24
B) 1.8
C) 3.6
D) 6.0

40. Suppose $X_1, X_2, \ldots, X_{10}$ are 10 independent $N(0, \sigma^2)$ variables. Consider now the following two lists of which items in List – 1 are statistics and items in List – 2 are some distributions.

List – 1	List – 2
i) $T_1 = \frac{1}{\sqrt{10}}\left\{\sum_{i=1}^{5} X_{2i-1} - \sum_{i=1}^{5} X_{2i}\right\}$	a) F(4, 4) distribution
ii) $T_2 = \left\{\sum_{i=1}^{5} X_{2i-1}^2\right\} \div \left\{\sum_{i=1}^{5} X_{2i}^2\right\}$	b) Students t-distribution with 4 df
iii) $T_3 = \left\{\sum_{i=1}^{5} X_{2i-1}\right\} \div \left\{\sum_{i=1}^{5} X_{2i}^2\right\}^{1/2}$	c) F(5, 5) distribution
	d) Standard normal distribution
	e) Students t-distribution with 5 df

Which of the following is then the correct match ?

A) (i) – (d), (ii) – (c), (iii) – (e)

B) (i) – (d), (ii) – (b), (iii) – (a)

C) (i) – (c), (ii) – (a), (iii) – (e)

D) (i) – (c), (ii) – (b), (iii) – (e)

41. The joint pdf of the bivariate random vector (X, Y) is given by

$$f(x, y) = \begin{cases} x + y \ , & 0 < x < 1, \quad 0 < y < 1 \\ 0 \ , & \text{otherwise} \end{cases}$$

Now consider List – 1 and List – 2 as given below :

List – 1	List – 2
i) Marginal pdf of X	a) $g_1(y) = \dfrac{x+y}{x+\frac{1}{2}}$, $0 < y < 1$
ii) Conditional pdf of Y given X = x	b) $g_2(x) = x$, $0 < x < 1$
iii) Correlation coefficient between X and Y	c) $g_3(x) = x + \frac{1}{2}$, $0 < x < 1$
	d) $g_4(y) = (x + y)/x$, $0 < y < 1$
	e) $1/11$
	f) $-\frac{1}{11}$

Then which of the following is the correct match ?

A) (i) – (b), (ii) – (d), (iii) – (f)
B) (i) – (c), (ii) – (a), (iii) – (f)
C) (i) – (c), (ii) – (a), (iii) – (e)
D) (i) – (b), (ii) – (a), (iii) – (e)

42. With usual notations, let (X, Y) follow a bivariate normal distribution $N(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Now consider the statements

i) $E(Y \mid X = x) = \mu_2 + \rho \dfrac{\sigma_2}{\sigma_1}(x - \mu_1)$ and

ii) $Var(Y \mid X = x) = \sigma_2^2 (1 - \rho^2)$.

Then which of the following is correct ?

A) (i) only
B) (ii) only
C) both (i) and (ii)
D) neither (i) nor (ii)

43. Let (1, 3), (3, 1), (2, 4), (4, 2) and (5, 5) be the five observations drawn from a bivariate normal distribution with ρ as the correlation coefficient. Then the value of students – t statistic for testing the null hypothesis $\rho = 0$ is equal to

A) $\frac{1}{\sqrt{8}}$
B) $\frac{\sqrt{3}}{2}$
C) $\sqrt{(5/3)}$
D) $\frac{2}{\sqrt{3}}$

44. Suppose $X_1, X_2, ..., X_n$ are the independent observations drawn from a population with mean μ and finite variance σ^2. Now consider the following statements on

$$T_n = \frac{2\sum_{r=1}^{n} r X_r}{n(n+1)}:$$

i) T_n is unbiased for μ

ii) T_n is a consistent estimator of μ

iii) T_n is a better estimator of μ than $\overline{X}$.

Then which of the following is correct ?

A) (i) and (ii) only
B) (i) and (iii) only
C) (ii) and (iii) only
D) all (i), (ii) and (iii)

45. Suppose 24, 20, 23, 28, 27, 30, 28, 20 are the observations drawn from $N(\mu, \sigma^2)$. Further it is known that $\mu \in (26, 32)$. Then which of the following is a more preferable estimate of μ ?

A) 25
B) 25.5
C) 26
D) 28.25

46. Observe the following statements :

i) Unbiasedness is invariant under transformations by continuous functions

ii) Consistency is invariant under continuous functional transformations

iii) UMVU estimation is invariant under continuous functional transformations

iv) Maximum likelihood estimation is invariant under continuous functional transformations

Now state which of the following is correct ?

A) (i) and (iii)
B) (ii) and (iv)
C) (i), (ii) and (iii)
D) (ii), (iii) and (iv)

47. Suppose $X_{n:n}$ is the n^{th} order statistic of a random sample of size n drawn from the uniform distribution over $(0, \theta)$. Now consider the following statements

i) $X_{n:n}$ is a complete sufficient statistic for θ

ii) $\frac{n}{n+1} X_{n:n}$ is the UMVU estimator of θ

iii) $X_{n:n}$ is the maximum likelihood estimator of θ

iv) $X_{n:n/2}$ is the method of moments estimator of θ

Then which of the following is correct ?

A) (i) and (iii)
B) (ii) and (iv)
C) (ii) and (iii)
D) (ii), (iii) and (iv)

48. A genetic trial results in n animals which are grouped into four sets with probabilities $\frac{1-p}{2}, \frac{p}{2}, \frac{p}{2}$ and $\frac{1-p}{2}$. The number of animals in the four groups are n_1, n_2, n_3 and n_4 respectively. If $0 < p < 1$, consider the following statements

i) Maximum likelihood estimator $\hat{p}$ of p is $\hat{p} = \frac{n_2 + n_3}{n}$

ii) Asymptotic variance of $\hat{p}$ is $\frac{p(1-p)}{n}$

Then which of the following is true ?

A) (i) only
B) (ii) only
C) both (i) and (ii)
D) neither (i) nor (ii)

49. If the genetic trial data as explained in question No. 48 is at hand, then the estimate p* of p obtained by modified minimum chi-square method is given by

A) $\frac{n_1 + n_4}{n}$
B) $\frac{n_2 + n_3}{n}$
C) $\frac{\frac{1}{n_1} + \frac{1}{n_4}}{\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \frac{1}{n_4}}$
D) $\frac{\frac{1}{n_2} + \frac{1}{n_3}}{\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \frac{1}{n_4}}$

50. Given a most powerful test φ of level α for testing a simple hypothesis H_0 against a simple alternative H_1 with power given by $E(\varphi / H_1) = \beta$. Then the test $\varphi^* = 1 - \varphi$ is a most powerful test for testing H_1 against H_0 and the level of the test is equal to

A) $1-\beta$
B) $1-\alpha$
C) α
D) β

51. Consider the following statements regarding MLR property of distributions :

i) Exponential family of distributions possess the MLR property.

ii) The family of distributions having MLR property is a bigger family than the exponential family of distributions.

iii) There is not any distribution other than those belonging to exponential class of distributions which possesses the MLR property.

Now state which of the following is correct ?

A) (i) and (ii)
B) (i) and (iii)
C) (i) only
D) (iii) only

52. Suppose for a data drawn from a population, three parameter Weibull distribution was fitted. Consequently the resulting table of observed and expected frequencies against the classes are given below.

Class	0 – 3	3 – 6	6 – 9	9 – 12	12 – 15	15 – 18	18 – 21
Observed frequency	20	16	14	11	9	6	4
Expected frequency	22	17	13	12	8	5	3

Then which of the following is the degrees of freedom of the chi-square goodness of fit statistic that one may compute ?

A) 3 B) 2 C) 6 D) 5

53. The blood bilirubin level of 8 patients before and after the administration of a medicine are given below.
(6.0, 6.2), (5.8, 3.6), (6.3, 6.8), (4.3, 5.2), (7.5, 6.0), (4.9, 3.8), (6.4, 5.8), (3.5, 1.9). Then to test H_0 : The treatment is not having any impact against. H_1 : Treatment is effective to reduce the bilirubin, by Wilcoxon matched pairs signed rank test, consider the following two lists of items.

List – 1	List – 2
i) Value of the test statistic T	a) 51
ii) E (T)	b) 7
iii) Var (T)	c) 18
	d) 12

Then which of the following is the correct match ?

A) (i) – (d), (ii) – (b), (iii) – (c)
B) (i) – (b), (ii) – (c), (iii) – (a)
C) (i) – (d), (ii) – (b), (iii) – (a)
D) (i) – (c), (ii) – (d), (iii) – (b)

54. Suppose there are two samples X = (1, 5, 7, 9, 15, 17, 21, 23) and Y = (2, 6, 10, 12, 18, 20, 26, 28, 32). Now for testing H_0 : The two samples are drawn from the same population using Wald-Wolfowitz run test, consider the following two lists of items :

List – 1	List – 2
i) Value of the test statistic d	a) $\frac{161}{17}$
ii) E (d)	b) 8
iii) Var (d)	c) $\frac{9 \times 127}{289}$
	d) 10

Then state which of the following is the correct match ?

A) (i) – (d), (ii) – (a), (iii) – (c)
B) (i) – (b), (ii) – (d), (iii) – (a)
C) (i) – (b), (ii) – (c), (iii) – (a)
D) (i) – (a), (ii) – (c), (iii) – (b)

55. In applying Mann-Whitney two sample test, if two samples each of size n are pooled and ordered giving R_1 and R_2 as the total ranks for the X observations (first sample) and Y observations (second sample) respectively and if u_{12} and u_{21} are the number of times X's proceeds Y and number of times Y's proceeds X's respectively, then consider the following relations :

i) $u_{12} = \frac{n(3n+1)}{2} - R_1$

ii) $u_{21} = \frac{n(3n+1)}{2} - R_2$

iii) $u_{12} + u_{21} = n.$

Now state which of the following is correct ?

A) only (i) B) only (ii) C) (i) and (ii) D) all (i), (ii) and (iii)

56. Given $X_1, X_2, \ldots, X_n$ are the independent observations drawn from a Bernoulli distribution with parameter θ. If we write $P = \left(\sum_{i=1}^{n} X_i\right) \big/ n$, then an asymptotic confidence interval for θ with confidence coefficient $1-\alpha$ using the upper $\left(\alpha/2\right)^{th}$ quantile $Z_{\alpha/2}$ of standard normal distribution is given by

A) $\left\{P - Z_{\alpha/2}\sqrt{\frac{\theta(1-\theta)}{n}}, P + Z_{\alpha/2}\sqrt{\frac{\theta(1-\theta)}{n}}\right\}$

B) $\left\{P - Z_{\alpha/2}\frac{\theta}{\sqrt{n}}, P + Z_{\alpha/2}\frac{\theta}{\sqrt{n}}\right\}$

C) $\left\{P - Z_{\alpha/2}\sqrt{\frac{P(1-P)}{n}}, P + Z_{\alpha/2}\sqrt{\frac{P(1-P)}{n}}\right\}$

D) $\left\{P - Z_{\alpha/2}\sqrt{\frac{P^2}{n}}, P + Z_{\alpha/2}\sqrt{\frac{P^2}{n}}\right\}$

57. Suppose (X_i, Y_i), i = 1, 2, . . ., n is a random sample of size n drawn from a bivariate normal distribution $N(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Define $D_i = X_i - Y_i$, i = 1, 2, . . ., n,

$$\bar{D} = \left(\sum_{i=1}^{n} D_i\right)\Big/ n,\ S_D^2 = \frac{1}{n-1}\sum_{i=1}^{n}(D_i - \bar{D})^2,\ \bar{X} = \left(\sum_{i=1}^{n} X_i\right)\Big/ n,\ \bar{Y} = \left(\sum_{i=1}^{n} Y_i\right)\Big/ n,$$

$S^2 = \sum_{i=1}^{n}(X_i - \bar{X})^2 + \sum_{i=1}^{n}(Y_i - \bar{Y})^2$ and $t_{\alpha/2}$ is the upper $\left(\alpha/2\right)^{th}$ quantile of students t-distribution with n – 1 degrees of freedom. Then the (1 – α) 100 % confidence interval for $\mu_1 - \mu_2$ is given by

A) $\left\{\bar{X} - \bar{Y} - t_{\alpha/2}\sqrt{\frac{2}{n}S^2},\ \bar{X} - \bar{Y} + t_{\alpha/2}\sqrt{\frac{2}{n}S^2}\right\}$

B) $\left\{\bar{D} - t_{\alpha/2}\sqrt{\frac{S_D^2}{n}},\ \bar{D} + t_{\alpha/2}\sqrt{\frac{S_D^2}{n}}\right\}$

C) $\left\{\bar{X} - \bar{Y} - t_{\alpha/2}\sqrt{\frac{2}{n}S_D^2},\ \bar{X} - \bar{Y} + t_{\alpha/2}\sqrt{\frac{2S_D^2}{n}}\right\}$

D) $\left\{\bar{D} - t_{\alpha/2}\sqrt{\frac{2S^2}{n}},\ \bar{D} + t_{\alpha/2}\sqrt{\frac{2S^2}{n}}\right\}$

58. Suppose the population distribution is $N(\mu, \sigma^2)$ in which σ^2 is known and the prior distribution of the unknown parameter μ is $N(\mu^*, \sigma^{*2})$. Then for $\frac{c}{b} = \left(\frac{\mu^*}{\sigma^{*2}} + \frac{n\bar{x}}{\sigma^2}\right)\left(\frac{1}{\sigma^{*2}} + \frac{n}{\sigma^2}\right)^{-1}$ and $\frac{1}{b} = \left(\frac{1}{\sigma^{*2}} + \frac{n}{\sigma^2}\right)^{-1}$, the posterior distribution of μ is

A) $N\left(\frac{c}{b}, \frac{1}{b}\right)$

B) Exponential with location parameter $\frac{c}{b}$ and scale parameter $\frac{1}{b}$

C) Gamma with scale parameter $\frac{1}{b}$ and shape parameter $\frac{c}{b}$

D) Logistic with location parameter $\frac{c}{b}$ and scale parameter $\frac{1}{b}$

59. Suppose the pdf of a random variable is given by

$$f(x) = \begin{cases} 3(1-x)^2, & 0 < x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Let 0.936, 0.875, 0.992 and 0.657 be four random numbers selected from the interval [0, 1]. Then which of the following shall be considered as the random sample of size 4 drawn from the above distribution ?

A) 0.6, 0.5, 0.8, 0.3
B) 0.312, 0.292, 0.331, 0.219
C) 0.064, 0.125, 0.008, 0.343
D) 0.936, 0.875, 0.992, 0.657

60. Complete enumeration on the mango yields of 216 trees in a farm gave a mean yield of 49 kg per tree with a variance of 150 kg. With simple random sampling how many trees would have been taken to estimate the mean yield within 10 % apart from a chance of 1 in 20 ?

A) 43
B) 22
C) 11
D) 10

61. With usual notations consider stratified random sampling involving K stratas yielding a sample mean $\overline{y}_{st}$. Then for a fixed total size n of the sample, Var $(\overline{y}_{st})$ becomes minimum if

A) $n_h = n \times \frac{N_h}{N}$
B) $n_1 = n_2 = \ldots = n_k = \frac{n}{k}$
C) $n_h = n \cdot \frac{W_h S_h}{\sum_{i=1}^{K} W_i S_i}, W_h = \frac{N_h}{N}$
D) $n_h = n \cdot \frac{S_h}{\sum_{i=1}^{K} S_i}$

62. With usual notations consider the following statements on ratio estimates and their variances for the population total Y, the population mean $\overline{Y}$ and the population ratio Y/X.

i) $\hat{Y}_R = \frac{\overline{y}}{\overline{x}} X, \text{Var}(\hat{Y}_R) = \frac{N^2(1-f)}{n(N-1)} \sum_{i=1}^{N} (y_i - Rx_i)^2$

ii) $\hat{\overline{Y}}_R = \frac{\overline{y}}{\overline{x}} \overline{X}, \text{Var}\left(\hat{\overline{Y}}_R\right) = \frac{(1-f)}{n(N-1)} \sum_{i=1}^{N} (y_i - Rx_i)^2$

iii) $\hat{R} = \frac{\overline{y}}{\overline{x}}, \text{Var}(\hat{R}) = \frac{(1-f)}{n\overline{X}^2 (N-1)} \sum_{i=1}^{N} (y_i - x_i)^2$

Now state which of the following is correct ?

A) (i) and (ii) only
B) (i) and (iii) only
C) (ii) and (iii) only
D) All (i), (ii) and (iii)

63. Suppose Y_1, Y_2, Y_3 are three independent stochastic variables with common variance σ^2 and means given by $E(Y_1) = \theta_1 + \theta_2$, $E(Y_2) = \theta_2 + \theta_3$, $E(Y_3) = \theta_1 + \theta_3$. Now consider the following statements involving the BLUE's $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_1 + \hat{\theta}_2 + \hat{\theta}_3$:

i) $\hat{\theta}_1 = \frac{1}{2}(Y_1 - Y_2 + Y_3)$

ii) $\hat{\theta}_2 = \frac{1}{2}(Y_1 - Y_2 - Y_3)$

iii) $\hat{\theta}_1 + \hat{\theta}_2 + \hat{\theta}_3 = \frac{1}{2}(Y_1 + Y_2 + Y_3)$

Now state which of the following is correct ?

A) (i) and (ii) only
B) (i) and (iii) only
C) (ii) and (iii) only
D) All (i), (ii) and (iii)

64. Consider the linear model $(Y, A\theta, \sigma^2 I)$ with an Assertion (A) and a Reason (R).

Assertion (A): The BLUE of any estimable function $b'\theta$ is uncorrelated with a linear function $l'Y$ belonging to the error.

Reason (R) : The coefficient vector l' of $l'Y$ is orthogonal with every column vector of A.

Now state which of the following is correct ?

A) Both A and R are true and R is the correct explanation of A

B) Both A and R are true, but R is not the correct explanation

C) A is true but R is false

D) Both A and R are false

65. There are two designs D_1 and D_2 of which D_1 is a CRD with treatments A, B, C, D and are replicated 4, 4, 3, 5 times respectively. D_2 is Graeco-Latin square design with six treatments. Then which of the following is correct ?

A) D_1 is a balanced design but D_2 is not balanced

B) D_2 is a balanced design but D_1 is not balanced

C) Both D_1 and D_2 are balanced designs

D) Neither D_1 nor D_2 is balanced

66. In a LSD with t-treatments, if we write R, C and E to denote the row, column and error mean squares respectively, then which of the following is the efficiency of LSD relative to RBD with Rows treated as blocks ?

A) $\frac{C+(t-1)E}{tE}$ B) $\frac{R+(t-1)E}{tE}$

C) $\frac{R+C+(t-1)E}{(t+1)E}$ D) $\frac{(t-1)R+E}{tE}$

67. Which of the following basic principles of experimental designs is/are not applied for a completely randomized design ?

A) Randomization B) Replication

C) Local control D) Randomization and Replication

68. In a RBD with 4 blocks and 6 treatments, one observation is missing. Then the SE of the mean of the treatment involving a missing value is given by

A) $\frac{\sigma}{2}$ B) $\frac{\sigma}{\sqrt{6}}$ C) $\frac{\sigma}{\sqrt{5}}$ D) $\sigma\sqrt{\frac{7}{20}}$

69. A design with 9 treatments and 12 blocks is given below.

Block No.	Treatments		
1	1	2	3
2	4	7	8
3	5	6	9
4	1	4	5
5	2	7	6
6	3	8	9

Block No.	Treatments		
7	1	7	9
8	2	5	8
9	3	4	6
10	1	6	8
11	2	4	9
12	3	5	7

Then the error degrees of freedom of the design is

A) 16 B) 22 C) 24 D) 18

70. A 2^5 experiment has to be laid out with reduced block size in three replications. For this the interactions ABC, ADE and ABE are confounded in all replications. Now consider the following two lists of items associated with the confounding system.

List – 1	List – 2
i) Treatments degrees of freedom	a) {(1), abd, ace, bcde}
ii) Error degrees of freedom	b) 24
iii) Intra block subgroup	c) BCDE, BC, DE, CE
iv) Other interactions confounded	d) 48
	e) {(1), ab, ae, be}
	f) BCDE, BD, ACD, CE

Which of the following is then the correct match ?

A) (i) – (b), (ii) – (d), (iii) – (a), (iv) – (f)

B) (i) – (d), (ii) – (b), (iii) – (e), (iv) – (c)

C) (i) – (b), (ii) – (d), (iii) – (e), (iv) – (c)

D) (i) – (d), (ii) – (b), (iii) – (e), (iv) – (f)

71. Given $X \sim N_p(\mu, \sigma^2 I_n)$ and P is a non-negative symmetric matrix of rank r.

Suppose $\frac{(X-\mu)' P(X-\mu)}{\sigma^2} \sim \chi^2(r)$. Now state which of the following is true ?

A) P is orthogonal

B) P is non-singular

C) P is such that trace P = r

D) P is idempotent

72. Suppose X and Y are independent p-variate random vectors such that $X + Y \sim N_p(\mu, \Sigma)$. Then which of the following is true ?

A) X and Y both need not be multivariate normally distributed

B) One of X or Y need not be multivariate normally distributed

C) X and Y are both multivariate and normally distributed with identical dispersion matrix

D) Both X and Y are distributed as multivariate normal with arbitrary mean vectors and dispersion matrices

73. Let $X_1, X_2, \ldots, X_N$ be N independent observations drawn from $N_P(\mu, \Sigma)$. Define

$$\overline{X} = \frac{1}{N}\sum_{\alpha=1}^{N} X_\alpha,\ A = \sum_{\alpha=1}^{N} X_\alpha X_\alpha^{'} - N\overline{X}\,\overline{X}^{'} \text{ and } T^2 = N(N-1)\left(\overline{X}-\mu_0\right)^{'} A^{-1}\left(\overline{X}-\mu_0\right).$$

Then to test $H_0 : \mu = \mu_0$ which of the following statistic with F(p, N – p) distribution is used ?

A) $\frac{(N-1)p}{N-p}T^2$

B) $\frac{(N-1)p}{N-p-1}T^2$

C) $\frac{N-p}{(N-1)p}T^2$

D) $\frac{N-p-1}{(N-1)p}T^2$

74. For a trivariate data with n = 27, the bivariate correlation coefficients are $r_{12} = \frac{1}{2}$, $r_{13} = \frac{1}{3}$ and $r_{23} = \frac{1}{6}$. Now consider the following two lists of items :

List – 1	**List – 2**
i) The value of the partial correlation coefficient $r_{12.3}$	a) $\sqrt{11/35}$
ii) The value of t-statistic for testing the significance of $\rho_{12.3}$	b) $11/2$
iii) The value of multiple correlation coefficient $R_{1.23}$	c) $\sqrt{8}/\sqrt{35}$
iv) The value of F-statistic for testing the significance of population multiple correlation coefficient	d) $8/\sqrt{3}$

Then which of the following is the correct match ?

A) (i) – (c), (ii) – (d), (iii) – (a), (iv) – (b)

B) (i) – (d), (ii) – (c), (iii) – (a), (iv) – (b)

C) (i) – (a), (ii) – (b), (iii) – (d), (iv) – (c)

D) (i) – (b), (ii) – (a), (iii) – (c), (iv) – (d)

75. Consider the transition probability matrix P given by

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$ and the following statements :

i) P is the Transition Probability Matrix of an irreducible Markov Chain (MC)

ii) All states of the MC associated with P are recurrent

iii) All states of the MC associated with P are of period four

Now state which of the following is correct ?

A) (i) and (ii) only
B) (ii) and (iii) only
C) All (i), (ii) and (iii)
D) (i) and (iii) only

76. Which of the following is a necessary and sufficient condition for a state i of a MC is recurrent ?

A) $\sum_{n=1}^{\infty} P_{ii}^{n} < \infty$
B) $\sum_{n=1}^{\infty} P_{ii}^{n} = \infty$
C) $\sum_{n=1}^{\infty} P_{ii}^{n} = 1$
D) $\sum_{n=1}^{\infty} P_{ii}^{n} < 1$

77. Let $\{X_t\}$ be a Poisson process. Then the time interval between the recurrence of successive Poisson events is distributed as

A) Poisson
B) Negative exponential
C) Uniform
D) Gamma

78. In (M/M/1) queue, let T_q denote the time spent waiting by a customer in the queue and let $W_q(t)$ denote the distribution function of T_q. Suppose $\left(\frac{1}{\lambda}\right)$ and $\left(\frac{1}{\mu}\right)$ are the mean inter arrival time and mean service time of the queue respectively. Then under steady state conditions and for $\rho = \frac{\lambda}{\mu}$ which of the following is correct ?

A) $W_q(t) = \begin{cases} 1-\rho, & t = 0 \\ 1-\rho e^{-\mu t}, & t > 0 \end{cases}$

B) $W_q(t) = \begin{cases} 1-\rho, & t = 0 \\ 1-e^{-(1-\rho)t}, & t > 0 \end{cases}$

C) $W_q(t) = \begin{cases} 1-\rho, & t = 0 \\ 1-\rho e^{-\rho t}, & t > 0 \end{cases}$

D) $W_q(t) = \begin{cases} 1-\rho, & t = 0 \\ 1-\rho e^{-\mu t(1-\rho)}, & t > 0 \end{cases}$

79. The prices of a commodity per unit during different quarters over the years 2010 – 2014 are given below :

Year	Price During Quarters			
	Q_1	Q_2	Q_3	Q_4
2010	51	69	58	91
2011	48	68	59	87
2012	40	58	61	93
2013	56	72	78	102
2014	50	68	84	107

Then which of the following represent the seasonal indices of the commodity ?

A) 49, 67, 68, 96
B) 100, 137, 139, 196
C) 70, 96, 97, 137
D) 100, 124, 128, 182

80. The prices and quantities of three commodities A, B and C for the years 2014 and 2015 are given below :

Commodity	2014		2015	
	Price p_0	Quantity q_0	Price p_1	Quantity q_1
A	3	6	4	12
B	3	8	8	16
C	4	2	6	4

Then the Fisher's ideal index number is equal to

A) 141
B) 200
C) 172
D) 400