A

16621 **120 MINUTES**

1. The number of one to one functions from a set A containing 4 elements to a set B containing 5 elements is
   A) 20 B) 24 C) 120 D) 125

2. Which of the following is a domain of the real valued function $y = \log(3x - x^2 - 2)$
   A) (1, 2) B) (1, 3) C) (2, 3) D) $(0, \infty)$

3. The values of x which satisfy the equation $|x^2 + 3x| + x^2 - 2 = 0$ are
   A) $-2, -2/3$ B) $-2, 1/2$ C) $2, -1/2$ D) $-2/3, 1/2$

4. The number of common tangents to the circles $x^2 + y^2 + 2x - 2y - 14 = 0$ and $x^2 + y^2 - 2x - 2y + 1 = 0$ is
   A) 1 B) 2 C) 4 D) 0

5. The coordinates of the vertex and the focus of the parabola $x^2 + 8x + 12y + 4 = 0$ are
   A) (– 4, 1) and (0, 3) B) (– 4, 1) and (–4, –2)
   C) (– 4, 1) and (0, –3) D) (– 4, 1) and(3, 0)

6. The direction cosines of the line joining the two points (4, –3, 3) and (6, –2, 1) are
   A) –2/3, –1/3, 2/3 B) –2/3, 1/3, 2/3
   C) 2/3, –1/3, 2/3 D) –2/3, 1/3, –2/3

7. The distance between the planes $2x - 3y + 6z + 12 = 0$ and $2x - 3y + 6z - 2 = 0$ is
   A) 1/2 B) 2 C) 7 D) 1/7

8. $\lim_{x\to 0} \frac{1}{x} \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) =$
   A) 0 B) 1 C) 2 D) 1/2

9. $\int_{-1}^{1} \frac{e^{2x} - 1}{e^{2x} + 1} dx =$
   A) –1/e B) 1/e C) 2/e D) 0

10. The area between the curves $x = 0$, $x = \pi/4$, $f(x) = \sin 2x$ and $g(x) = \cos 2x$ is
    A) $2\sqrt{2} + 1$ B) $2\sqrt{2} - 1$ C) $\sqrt{2} + 1$ D) $\sqrt{2} - 1$

11. The points at which the tangents to the curve $y = \frac{x^2 + 1}{x}$ are parallel to the X-axis are
    A) (1, –2) and (1, 2) B) (– 1, 2) and (1, 2)
    C) (1, 2) and (– 1, –2) D) (–1, –2) and (–1, 2)

12. There are five black balls and five red balls marked 1, 2, 3, 4, 5. The number of ways in which we can arrange these balls in a row so that neighbouring balls are of different colours is

A) $2(5!)$ B) $2(5!)^2$ C) $5!$ D) $(5!)^2$

13. A speaks the truth in 60% cases and B speaks truth in 70% cases. The probability that they will contradict each other when describing a single event is

A) 0.46 B) 0.42 C) 0.54 D) 0.3

14. $\lim_{x \to 2} \dfrac{e^x - e^2}{x - 2} =$

A) 1 B) e^2 C) $e^2 - 1$ D) $1 - e^2$

15. Let $f(x) = x \sin(1/x)$ for $x > 0$. Then which of the following is true?

A) $f(x) = 0$ has no solution in $(0, \infty)$

B) $f(x) = 0$ has exactly one solution in $(0, \infty)$

C) $f(x) = 0$ has infinitely many solutions in $(0, \infty)$

D) $f(x)$ is strictly increasing in $(0, \infty)$

16. Let $f_n(x) = x^n \sin \pi x$ for $0 \le x \le 1$ and let $f(x) = \lim_{n\to\infty} f_n(x)$. Then $f(1/2) =$

A) 0 B) 1

C) $\dfrac{1}{2}$ D) $\dfrac{\pi}{2}$

17. Which of the following statements are true about the functions?

$$f(x) = \begin{cases} x(1-x)\sin(1/x) & \text{for } x \neq 0 \\ 1 & \text{otherwise} \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x(1-x)\sin(1/(1-x)) & \text{for } x \neq 1 \\ 1 & \text{otherwise} \end{cases}$$

A) f and g are continuous at 0

B) f and g are continuous at 1

C) f is continuous at 0 and g is continuous at 1

D) f is continuous at 1 and g is continuous at 0

18. Let $f(x) = \begin{cases} \dfrac{x^2 + 2y^2}{x + y} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{otherwise} \end{cases}$

Then the directional derivative of f at (0, 0) along u = (1, 1) is

A) 0 B) 1

C) 3/2 D) 3

19. Which of the following statements are true about the functions f, g and h defined on [0, 1] where $f(x) = \sin x$,

$$g(x) = \begin{cases} x \sin(1/x) \text{ for } x \neq 0 \\ 0 \text{ otherwise} \end{cases} \quad \text{and} \quad h(x) = \begin{cases} \sin(1/x) \text{ for } x \neq 0 \\ 0 \text{ otherwise} \end{cases}$$

A) f and g are of bounded variation
B) f and h are of bounded variation
C) g and h are of bounded variation
D) f is of bounded variation and h is not bounded variation

20. Let $f(x) = 2x$ and $\alpha(x) = \cos x$. Then $\int_0^{\pi/2} f \, d\alpha =$

A) 0 B) 1 C) 2 D) –2

21. Let F be a non measurable subset of the real line **R** and G be a measurable subset of **R** of measure zero. Then which of the following is a non measurable subset of **R**. Here for any set A, A^c represents the complement of A

A) F U G B) $F \cap G$ C) $F^c \cup G^c$ D) $F^c \cap G$

22. Let $f(x) = \begin{cases} x & \text{for } 0 \le x \le 1 \\ 2x - 1 & \text{for } 1 \le x \le 2 \end{cases}$, $g(x) = \begin{cases} 1 & \text{for } 0 \le x \le \frac{1}{2} \\ 2 & \text{for } \frac{1}{2} \le x \le 1 \\ 1 & \text{for } 1 \le x \le 3 \end{cases}$ and

$h(x) = \begin{cases} x + 1 & \text{for } 0 \le x \le 1 \\ 2 & \text{for } 1 \le x \le 3 \end{cases}$. Then which of the following is not a simple function?

A) f o g B) g o f C) h o f D) h o g

23. Which among the following statements are true about the complex number z = 1 + 2i.

(i) $|e^z| = e$ (ii) $|e^z| = |e^{-z}|$ (iii) $|e^{z^2}| = e^{-3}$

A) (i) and (ii) B) (i) and (iii) C) (ii) and (iii) D) (i) only

24. The set $K = \{\cos\theta + i\sin\theta : 0 \le \theta \le \pi\}$ represents

A) The upper half of the unit circle
B) The upper half of the unit disk
C) The unit circle
D) The unit disk

25. $\int_{|z|=2} \frac{e^z}{(z-1)^2} dz =$

A) $2\pi i$ B) $2\pi ie$ C) $4\pi i$ D) $4\pi ie$

26. Which of the following statements about the function $f(z) = e^{1/z}$ are true. Here D denotes the open unit disk $|z| < 1$?

(i) $|f(z)| > 1$ for all $z \in D$ (ii) there exists $z \in D$ such that $|f(z)| > 1$

(iii) there exists $z \in D$ such that $|f(z)| < 1$

A) (i) and (ii) only B) (i) and (iii) only

C) (ii) and (iii) only D) (i), (ii) and (iii)

27. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n^2 z^n}{n!}$ is

A) 0 B) 1

C) 2 D) ∞

28. Let **N** denote the set of natural numbers and let * denote the binary operation on **N** defined by x * y = LCM of x and y for all x, y ∈ **N**. Then which of the following is not a solution of the equation x * y = 6 * y

A) x = 2, y = 9 B) x = 3, y = 8

C) x = 4, y = 9 D) x = 4, y = 12

29. Let G be a cyclic group of order 7. Then the number of automorphisms of G is

A) 1 B) 2

C) 6 D) 7

30. Which of the following pairs of groups are isomorphic to each other?

A) $\mathbf{Z}_{100}$ and $\mathbf{Z}_{50}$ X $\mathbf{Z}_2$ B) $\mathbf{Z}_{72}$ and $\mathbf{Z}_{24}$ X $\mathbf{Z}_3$

C) $\mathbf{Z}_{50}$ and $\mathbf{Z}_{10}$ X $\mathbf{Z}_5$ D) $\mathbf{Z}_{36}$ and $\mathbf{Z}_9$ X $\mathbf{Z}_4$

31. The order of the element (1 2) (3 4 5) in the symmetric group S_5 is

A) 2 B) 3 C) 5 D) 6

32. Let G be a group and H, K be subgroups of G such that H is normal in G and $H \cap K = \{1\}$ where 1 is the identity of G. Then which of the following is not a necessary property of H and K?

A) HK = KH B) hk = kh for all h∈ H, k ∈ K

C) Hx = xH for every x ∈ K D) $hk = h_1k_1 \Rightarrow h = h_1$ and $k = k_1$

33. The commutator subgroup of the symmetric group S_3 is isomorphic to

A) S_3 B) $\mathbf{Z}_3$

C) $\mathbf{Z}_2$ D) (1)

34. Let R be the ring of all 2 x 2 matrices over the reals. Then which of the following matrices are in the centre of R?

$X = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad Y = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

A) X only B) Y only
C) X and Y only D) Y and Z only

35. Let $\mathbf{Z}_{54}$ be the ring of integers modulo 54. Then which of the following is a set of zero divisors of $\mathbf{Z}_{54}$

A) {2, 3, 4, 5} B) {3, 4, 5, 6}
C) {3, 5, 7, 9} D) {2, 4, 6, 8}

36. Let G_1 = Aut $\mathbf{Q}(\sqrt{2})$ and G_2 = Aut $\mathbf{Q}(\alpha)$ where α is the real cube root of 2. Then which of the following is true?

A) G_1 and G_2 are isomorphic B) $|G_1| = |G_2|$
C) $|G_1| > |G_2|$ D) $|G_2| = 3$

37. Let F be a field of characteristic 2. Consider the following statements
(i) $(a+b)^2 = a^2 + b^2$ for all a, b ∈ F (ii) $(a+b)^3 = a^3 + b^3$ for all a, b ∈ F
(iii) $(a+b)^4 = a^4 + b^4$ for all a, b ∈ F. Then which among these are true?

A) (i) and (ii) only B) (i) and (iii) only
C) (ii) and (iii) only D) (i), (ii) and (iii)

38. Which of the following is not a splitting field over the rationals **Q**?

A) $\mathbf{Q}(\sqrt{2})$
B) **Q** (α) where α is the real cube root of 2
C) **Q** (ω) where ω is a non real cube root of unity
D) $\mathbf{Q}(\sqrt{2}, \sqrt{3})$

39. Let F be the field of rational functions over the field **Q**. Then which of the following statements are true?

(i) [F : **Q**] is infinite
(ii) [K : **Q**] is infinite for all subfields K of F with K ≠ **Q**
(iii) There exists a subfield K of F such that [K : **Q**] = 2

A) (i) and (ii) only B) (i) and (iii) only
C) (i) only D) (ii) only

40. Let $\mathbf{Z}_2(\alpha)$ be the extension of the field $\mathbf{Z}_2$ where α is a zero of the polynomial $x^3 + x^2 + 1$. Then $\frac{1}{\alpha}$ =

A) $1 + \alpha$ B) $1 + \alpha^2$ C) $1 + \alpha^3$ D) $\alpha + \alpha^2$

41. Which of the following is a nilpotent matrix?

A) $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ D) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

42. Let A be a 3 x 3 invertible matrix such that $A^3 - A = 0$. Then $A^{-1} =$

A) A B) A + 1 C) A – 1 D) $A^2 - 1$

43. Which among the following systems of equations are consistent/ inconsistent?

(i) $2x+y+z = 1$, $3x+y+z = 0$, $x+ y+z = 1$
(ii) $2x+2y+z = 2$, $3x+3y+2z =1$, $x+ y+z = 2$
(iii) $2x+2y+z = 1$, $3x+3y+2z = 2$, $x+y+z = 1$

A) (i) and (ii) are consistent
B) (i) and (iii) are consistent
C) (i) is consistent and (ii) is inconsistent
D) (i) is inconsistent and (iii) is consistent

44. Consider the linear independence of the following subsets of $\mathbf{R}^3$ and choose the correct statement

(i) {(1, 1, 1), (1, 1, 0), (1, 0, 0)} (ii) {(1, 1, 2), (1, 1, 3), (1, 1, 4)}

A) (i) and (ii) are linearly independent
B) (i) and (ii) are linearly dependent
C) (i) is linearly independent and (ii) is linearly dependent
D) (i) is linearly dependent and (ii) is linearly independent

45. Let $\{\upsilon_1, \upsilon_2, \upsilon_3\}$ be a basis of a vector space V over the reals. Then which of the following is also a basis of V?

A) $\{\upsilon_1 + \upsilon_2 + \upsilon_3, \upsilon_1 + \upsilon_2, \upsilon_1 - \upsilon_2\}$
B) $\{\upsilon_1 + \upsilon_2 + \upsilon_3, \upsilon_1 + 2\upsilon_2 + \upsilon_3, \upsilon_1 + 3\upsilon_2 + \upsilon_3\}$
C) $\{\upsilon_1 + \upsilon_2 + \upsilon_3, \upsilon_1 + \upsilon_2 - \upsilon_3, \upsilon_1 + \upsilon_2 - 2\upsilon_3\}$
D) $\{\upsilon_1 + \upsilon_2 + \upsilon_3, \upsilon_1 - \upsilon_2 + \upsilon_3, \upsilon_1 + \upsilon_3\}$

46. Let W_1, W_2 be subspaces of a vector space V and let $V = W_1 \oplus W_2$. Let $\{\upsilon_1, \upsilon_2\}$ be a basis of W_1 and $\{u_1, u_2\}$ be a basis of W_2. Then which of the following is not a basis of V?

A) $\{\upsilon_1 + u_1, \upsilon_2 + u_1, \upsilon_1, \upsilon_2\}$
B) $\{\upsilon_1 + u_1, \upsilon_2 + u_1, u_1, u_2\}$
C) $\{\upsilon_1 + u_1, v_2 + u_2, \upsilon_1, v_2\}$
D) $\{\upsilon_1 + u_2, \upsilon_2 + u_1, u_1, u_2\}$

47. Which of the following is true about the lines?
$L_1 = \{(x,y) : 2x + 3y = 0\}$, $L_2 = \{(x,y) : 2x + 3y = 1\}$, $L_3 = \{(x,y) : 2x - 3y = 0\}$

A) L_1 and L_2 are subspaces of $\mathbf{R}^2$
B) L_2 and L_3 are subspaces of $\mathbf{R}^2$
C) L_1 and L_3 are subspaces of $\mathbf{R}^2$
D) L_1, L_2 and L_3 are subspaces of $\mathbf{R}^2$

48. Consider the linear transformation $f : \mathbf{R}^3 \to \mathbf{R}^2$ given by $f(x, y, z) = (x + y, z)$. Then the dimension of the null space of f is

A) 0 B) 1
C) 2 D) 3

49. Which of the following is true about the linear transformations f, $g : \mathbf{R}^3 \to \mathbf{R}^3$ given by $f(x, y, z) = (x + y, x - y, x - z)$ and $g(x, y, z) = (x - y, y - z, z - x)$

A) f and g are invertible
B) f and g are non invertible
C) f is invertible and g is non invertible
D) f is non invertible and g is invertible

50. Let $f : \mathbf{R}^3 \to \mathbf{R}^3$ be a linear transformation which is represented by the matrix
$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Then which of the following is also a matrix representation of f?

A) $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

C) $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ D) $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

51. Let f be a linear functional on $\mathbf{R}^2$ for which $f(1,1) = 1$ and $f(2,1) = 0$. Then $f(2, 3) =$

A) 0 B) 1
C) 3 D) 4

52. Which of the following is an eigen vector of $f : \mathbf{R}^3 \to \mathbf{R}^3$ given by $f(x, y, z) = (x + y, x - y, z)$?

A) (1, 0, 0) B) (0, 1, 1)
C) (0, 0, 1) D) (1, 1, 0)

53. Which of the following is a diagonalizable matrix?

A) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

C) $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

54. The value of n for which $\emptyset(n) \neq n/2$ where $\emptyset$ is the Euler function, is

A) 2 B) 4 C) 6 D) 8

55. The linear congruence $4x \equiv 3 \pmod 5$ has

A) Exactly one solution (mod 5)
B) Two solutions (mod 5)
C) Three solutions (mod 5)
D) No solution

56. For any prime p which one of the following is true?

A) $p! \equiv -1 \pmod p$ B) $p! \equiv 1 \pmod p$
C) $(p + 1)! \equiv 1 \pmod p$ D) $(p - 1)! \equiv -1 \pmod p$

57. The number of values of x which simultaneously satisfy the system of congruences $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 4$, $x \equiv 3 \pmod 5$ is

A) 0 B) 1
C) 2 D) infinite

58. The differential equation of a family of circles passing through the origin with its centre on the x – axis is

A) $2y\frac{dy}{dx} = y^2 - x^2$ B) $2xy\frac{dy}{dx} = y^2 - x^2$

C) $2y\frac{dy}{dx} = x^2 - y^2$ D) $2xy\frac{dy}{dx} = x^2 + y^2$

59. The solution of the equation $\frac{dy}{dx} + y = \frac{1}{1 + e^{2x}}$ is

A) $y = e^{-x} \tan^{-1} (e^{x}) + ce^{-x}$ B) $y = e^{x} \tan^{-1} (e^{-x}) + ce^{x}$
C) $y = e^{-x} \tan^{-1} (e^{-x}) + ce^{-x}$ D) $y = e^{x} \tan^{-1} (e^{x}) + ce^{x}$

60. The number of regular singular points on the x-axis for the equation

$x^2 (x^2 - 1)^2 y'' - x (1 - x) y' - 2y = 0$ is

A) 1 B) 2 C) 3 D) 0

61. The 3rd degree Legendre polynomial $P_3(x)$ is

A) $5x^3 - 3x^2$ B) $2(5x^3 + 3x^2)$

C) $\frac{x}{2}(5x^2 - 3)$ D) $2x(5x^2 - 3)$

62. If $J_p(x)$ is the Bessel function of order p, then $\int x^{-p} J_{p+1}(x)dx$ is

A) $x^{-p} J_p(x) + c$ B) $-x^{-p} J_p(x) + c$

C) $-x^{p-1} J_p(x) + c$ D) $x^{p-1} J_{p-1}(x) + c$

63. The integral of the equation $(4x + yz)\,dx + (xz - 2y)\,dy + (xy + 4z)\,dz = 0$ is

A) $2x^2 + y^2 - 2z^2 + xyz = c$ B) $2x^2 - 2y^2 + 4z^2 - xyz = c$

C) $2x^2 - y^2 + 2z^2 - xyz = c$ D) $2x^2 - y^2 + 2z^2 + xyz = c$

64. The characteristic curves of the differential equation $2x\,u_y + 2y\,u_x = u$, where $u = u(x, y)$ is a function of x and y, are a family of

A) Circles B) Ellipses

C) Straight lines D) Hyperbolas

65. If u (x, t) satisfies the equation

$u_{xx} - u_{tt} = 0 \quad , \quad -\infty < x < \infty,\ t > 0$

$u(x, 0) = 2x \quad , \quad u_t(x, 0) = 0$

then u (4, 2) is

A) 8 B) 6 C) 4 D) 2

66. The equation $u_{xx} + xu_{yy} = 0$ is

A) Hyperbolic for all x

B) Elliptic for all x

C) Hyperbolic for x < 0 and elliptic for x > 0

D) Hyperbolic for x > 0 and elliptic for x < 0

67. Let X = {1, 2, 3, 4, 5} and τ be the topology on X given by

$\tau = \{x, \phi, \{1, 2\}, \{1, 2, 3\}\}$. Let

A = {3, 4}. Then $\overline{A}$ =

A) {3, 4} B) {3, 4, 5} C) {2, 3, 4} D) {4, 5}

68. Let X be the set of all real numbers and τ be the cofinite topology on X. Then which of the following sequence (a_n) is a non convergent sequence in X.

A) $a_n = \begin{cases} 0 \text{ if } n \text{ is even} \\ 1 \text{ if } n \text{ is odd} \end{cases}$ B) $a_n = \frac{n}{n+1}$

C) $a_n = \frac{n^2}{n+1}$ D) $a_n = \frac{n+1}{2}$

69. Let X be the set of all real numbers and d be the metric on X x X defined by d (x, y) = $|x_1 - y_1| + |x_2 - y_2|$ where x = (x_1, x_2) and y = (y_1, y_2). Then which of the following is a point outside the closed unit ball centered at (0, 0)?

A) $({}^{1}/_{2}, {}^{1}/_{2})$ B) $({}^{1}/_{2}, {}^{1}/_{3})$

C) $({}^{1}/_{\sqrt{2}}, {}^{1}/_{\sqrt{2}})$ D) $\left(\frac{1}{2}, \frac{1}{2\sqrt{2}}\right)$

70. Let **R** be the metric space of real numbers with usual metric. Consider the following statements

(i) Every finite subset of **R** is a closed set
(ii) Every infinite subset of **R** is an open set
(iii) Every non empty open subset of **R** is an infinite set

Which among these statements are correct?

A) (i) and (ii) only B) (i) and (iii) only
C) (ii) and (iii) only D) (i), (ii) and (iii)

71. Which of the following is a sub base for the usual topology of the real line?

A) $\{(a, \infty) : a \in \mathbf{R}\}$
B) $\{(a, \infty) : a \geq 0\} \cup \{-\infty, b) : b \leq 0\}$
C) $\{(a, \infty) : a \leq 0\} \cup \{-\infty, b) : b \geq 0\}$
D) $\{(a, \infty) : a \in \mathbf{R}\} \cup \{-\infty, b) : b \in \mathbf{R}\}$

72. For a subset A of a topological space let A^0 denote the interior of A and $\overline{A}$ denote the closure of A. Then which of the following is true about a non-empty subset A of X?

A) $(\overline{A})^0 = A^0$ B) $\overline{(A^0)} = \overline{A}$
C) $A^0 = X - (\overline{X - A})$ D) $(\overline{A}) = A \cup (X - A^0)$

73. Which of the following pairs of topological spaces are homeomorphic. Here **R** is the real line and **Q** is the subspace of rationals

A) **R** and $\mathbf{R}^2$ B) **Q** and **R** \ **Q**
C) **R** and (0, 1) D) [0, 1] and (0, 1)

74. Let X be the normed linear space R^2 with $\| \|_1$. If A ε B L(X) is represented by the matrix $\begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$ then $\|A\|$ is

A) 2 B) 3 C) 4 D) 7

75. Let X be the normed space R^2 with $\| \|_\infty$ and Y = $\{(x(1), x(2)) \in R^2 : x(1)+x(2)=0\}$ If x = (–1, –1) then d (x,Y), where d (x,Y) is the distance of x from Y, is

A) 0 B) 1 C) 2 D) $\sqrt{2}$

76. Let C_{00} = {x ε ℓ^{∞} : all but finitely many x(j)'s are zero} and a = $(1, \frac{1}{2}, \frac{1}{3}, \ldots)$. If f is any bounded linear functional on ℓ^{∞}, then which one of the following cannot hold?

A) f (a) = 0 but $f \neq 0$ on C_{00} B) $f = 0$ on C_{00} but $f(a) \neq 0$

C) f (a) $\neq 0$ and f $\neq 0$ on C_{00} D) f (a) = 0 and f = 0 on C_{00}

77. Let X be the inner product space R^2 over the real field R and let a = (1, 1). Then which one of the following set is orthogonal to a?

A) $\{(x(1), x(2)) \varepsilon R^2 : x(1) - x(2) = 0\}$

B) $\{x \varepsilon R^2 : \|x\| = 1\}$

C) $\{(x(1), x(2)) \varepsilon R^2 : x(1) + x(2) = 0\}$

D) $\{x \varepsilon R^2 : \|x\| = \|a\|\}$

78. Let X = R^3 and Y = R^2 be the normed linear spaces with $\| \quad \|_1$ and let F : X → Y be defined by F (x(1), x(2), x(3)) = (x(1), x(3)) for (x(1), x(2), x(3)) ε X. Then which one of the following is not true?

A) F is linear and continuous

B) F is linear and open

C) The set {(x, F(x)): x ε X} is closed in X x Y

D) One of the above is wrong

79. Let ℓ^2 be the Hilbert space of all square summable sequences of real numbers over the real field R. Let f be the linear functional on ℓ^2 defined by f (x(1), x(2), …) = x(1) – x(2) + x(3) – x(4) for all x = (x(1), x(2), …) εℓ^2. Then $\|f\|$ is

A) 2 B) $\sqrt{2}$ C) 4 D) 1

80. Let H be the Hilbert space $L^2 [-\pi, \pi]$. If $u_n = \frac{e^{int}}{\sqrt{2\pi}}$, n = 1, 2, 3, …, then which one of the following is true?

A) $\{u_n : n = 1, 2, \ldots\}$ is a maximal orthonormal set in H

B) If x ε H, then $x = \sum_{n=1}^{\infty} < x, u_n > u_n$

C) If x ε H, then $\|x\|^2 \geq \sum_{n=1}^{\infty} |< x, u_n >|^2$

D) If x ε H and $< x, u_n > = 0$ for all n, then x = 0

****