1. Given the interval I = [0,1] and a subset $A \subset I$. The function f: $A \rightarrow I$ is such that f(A) = C where C is the Cantor set. Then which of the following is true?
   A) A is countable
   B) A is uncountable
   C) A = I
   D) Such a function f does not exist

2. Let X and Y be two countable sets and consider the following statements:
   (i) $X \cap Y$ is countable,
   (ii) $X \cup Y$ is countable and
   (iii) The cartesian product XxY is countable.

   State which of the following is true?
   A) Only (i) is true
   B) Only (ii) is true
   C) (i) and (ii) are true but (iii) is not true
   D) (i), (ii) and (iii) are true

3. Suppose $\{S_n\}$is a sequence, where $S_n = \frac{1}{n+1} + \frac{1}{n+2} + ---- + \frac{1}{n+n}$. Consider the following statements:
   (i) $\{S_n\}$ is monotone decreasing
   (ii) $\{S_n\}$ is monotone increasing
   (iii) $\{S_n\}$ is bounded above
   (iv) $\{S_n\}$ is neither monotone increasing nor monotone decreasing.

   Now state which of the following is true?
   A) Only (i) and (iv)
   B) Only (ii) and (iv)
   C) Only (iii) and (iv)
   D) Only (ii) and (iii)

4. Suppose $\sum_{n=1}^{\infty} a_n$ is an absolutely convergent series of real numbers and $\{b_n\}$ is a bounded sequence of real numbers. Then
   A) $\sum_{n=1}^{\infty} a_n b_n$ is absolutely convergent
   B) $\sum_{n=1}^{\infty} b_n$ is absolultely convergent
   C) $\sum_{n=1}^{\infty} a_n b_n$ diverges
   D) $\sum_{n=1}^{\infty} a_n b_n$ need not be absolutely convergent

5. Consider the function f(x) = |x|, for $x \in R$. Then
   A) f is not continuous at x = 0, but differentiable at x = 0
   B) f is continuous at x = 0,but not differentiable at x = 0
   C) f is both continuous and differentiable at x = 0
   D) f is neither continuous nor differentiable at x = 0

6. Given d_1 and d_2 are two metrics defined on a set X. If $d_3 = d_1 + d_2$, $d_4 = \text{Max}(d_1,d_2)$, $d_5 = \frac{d_1}{1+d_1}$, then state which of the following is correct?

A) Only d_3 is a metric on X
B) Only d_4 is a metric on X
C) All d_3, d_4 and d_5 are metrics on X
D) d_3 and d_4 are metrics while d_5 is not a metric on X

7. Suppose (X, d) is a metric space and consider the following statements:
(i) Countable union of open sets of X is open
(ii) Countable union of closed sets of X is closed
iii) Countable intersection of open sets is open
iv) Countable intersection of closed sets is closed.

Then state which of the following is correct?

A) Only (ii) B) Only (iii)
C) Both (ii) and (iii) D) Both (i) and (iv)

8. Suppose A is a class of disjoint open intervals of the real line R. Then the class A is

A) Collection of finite number of sets
B) At most a countable collection of sets
C) Uncountable collection of sets
D) Collection of either countable or uncountable collection of sets

9. If z is a complex number and $\bar{z}$ is the complex conjugate of z, then the limit of the function $f(z) = z / \bar{z}$ is equal to

A) 1 B) – 1
C) 0 D) The limit does not exist

10. Suppose $f(z) = U(x,y) + iV(x,y)$ and $f'(z_0)$ exists where $z_0 = (x_0, y_0)$. If $U_x(x_0, y_0)$ and $V_x(x_0, y_0)$ are the first order partial derivatives of U and V with respect to x and $U_y(x_0,y_0)$ and $V_y(x_0, y_0)$ are the first order partial derivatives of U and V with respect to y, then state which of the following is true?

A) $U_x(x_0, y_0) = V_y(x_0, y_0)$ and $U_y(x_0, y_0) = -V_x(x_0, y_0)$
B) $U_x(x_0, y_0) = -V_y(x_0, y_0)$ and $U_y(x_0, y_0) = V_x(x_0, y_0)$
C) $U_x(x_0, y_0) = V_x(x_0, y_0)$ and $U_y(x_0, y_0) = -V_y(x_0, y_0)$
D) $U_x(x_0, y_0) = V_x(x_0, y_0)$ and $U_y(x_0, y_0) = V_y(x_0, y_0)$

11. Suppose f(z) is analytic everywhere within and on a simple closed contour C and z_0 is any point interior to C. Then $f(z_0)$ is equal to

A) $\frac{1}{2\pi i}\int_C (z-z_0)\, f(z)dz$ B) $\frac{1}{2\pi i}\int_C \frac{f(z)dz}{(z-z_0)}$

C) $\frac{1}{2\pi}\int_C (z-z_0)\, f(z)dz$ D) $\frac{1}{2\pi}\int_C \frac{f(z)dz}{(z-z_0)}$

12. Suppose x_1, x_2, x_3 are the vectors of a basis set of R^3. Consider now the following linear combinations of x_1, x_2, x_3:

$y_1 = x_1 + x_2 + x_3$
$y_2 = x_1 - x_2 + x_3$
$y_3 = 2x_1 + x_2 - x_3$
$y_4 = x_1 - 4x_3$

Then the dimension of the vector space generated by the vectors y_1, y_2, y_3, y_4 is equal to:

A) 1 B) 2 C) 3 D) 4

13. The characteristic roots of an idempotent matrix are

A) all 0 B) all 1
C) zeros and ones D) all rationals

14. If A is a matrix of order n x n and $|A|$ is its determinant then which of the following is not true?

A) $|A|$ is the product of the characteristic roots of A
B) If A is non singular then all of its characteristic roots are non zeros
C) The characteristic roots of a diagonal matrix are its diagonal elements
D) The characteristic roots of an upper triangular matrix are different from the matrix obtained by replacing the elements above its diagonal elements by zeros

15. The value of 'a' for which the vectors (1,3,–2), (2,5,0) and (–3,a,–5) in R^3 are linearly dependent is

A) $\frac{5}{2}$ B) $-\frac{11}{4}$ C) $-\frac{25}{4}$ D) 0

16. In order that the matrix given by

$$A = \begin{pmatrix} 0 & 2m & n \\ \ell & m & -n \\ \ell & -m & n \end{pmatrix}$$

is orthogonal, the values of ℓ, m, n in order are given by

A) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{3}}$ B) $\frac{1}{2}, \frac{1}{2}, 1$
C) –1, 2, –1 D) –2, 1, 1

17. If A and B are non-empty sets, then the lim inf E_n = E and lim sup E_n = F of the sequence of sets $\{E_n\}$ defined as $E_n = \begin{cases} A, \text{if n is odd} \\ B, \text{if n is even} \end{cases}$ are given by

A) E = A, F = B B) $E = A \cap B$, $F = A \cup B$
C) $E = A \cup B$, $F = A \cup B$ D) $E = A \cap B$, $F = A \cap B$

18. Given E and F are two σ fields of subsets of the set X. Then which of the following is true?

A) Both $E \cup F$ and $E \cap F$ are σ - fields
B) Neither $E \cup F$ nor $E \cap F$ is a σ - field
C) $E \cup F$ is a σ - field but $E \cap F$ is not a σ - field
D) $E \cup F$ is not necessarily a σ - field but $E \cap F$ is a σ - field

19. Given B is the Borel field. Let F_1, F_2 and F_3 be the σ - fields generated by (i) the class of all open intervals of the type (a,b) (ii) the class of all half open intervals of the type (a,b] and (iii) the class of all closed intervals of the [a,b] of the real line R respectively. Then which of the following is true?

A) $F_1 \subset F_2 \subset F_3 \subset B$ B) $B \subset F_1 \subset F_2 \subset F_3$
C) $B \neq F_1 = F_2 = F_3$ D) $F_1 = F_2 = F_3 = B$

20. If is the Lebesgue measure and A is the set of all rationals in the interval [–2, 2] then (A) is equal to

A) 4 B) 2
C) 0 D) A is not Lebesgue measurable

21. If the Stieltjes measure function F(x) is defined by

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-x}, & x \geq 0 \end{cases}$$

and if A = [–2, 2] , then the Lebesgue – Stieltjes measure of A is equal to

A) 4 B) $\frac{e^2 - 1}{e^2}$ C) $\frac{e - 1}{e}$ D) 2

22. The value of the integral $\int_0^2 |2x - 1| dx$ is equal to

A) $\frac{5}{2}$ B) $\frac{1}{4}$ C) $\frac{9}{4}$ D) $\frac{3}{2}$

23. If A and B are events such that P(A) = 1 = P(B), then P $(A \cap B)$ equals

A) 1
B) 1 only when A and B are independent events
C) a positive real number less than 1
D) a real number less than 1

24. Consider a random experiment with possible outcomes:
$a_1, a_2, \ldots, a_N$ in which a_{j+1} is twice as likely as a_j for j=1, 2, …., N-1.
Let A = $\{a_1, a_2, \ldots, a_k\}$ for k < N. Then P(A) is equal to:

A) $\frac{k}{N}$ B) $\frac{2^k}{2^N}$ C) $\frac{k^2}{N^2}$ D) $\frac{2^k - 1}{2^N - 1}$

25. Consider an experiment of tossing two unbiased coins. Then the probability of two heads given a head on the first coin is:

A) $\frac{1}{4}$ B) $\frac{1}{2}$

C) 1 D) $\frac{1}{3}$

26. First of two Urns contains 7 white and 10 black balls, while the second contains 5 white and 12 black balls. A person chooses an urn at random and draws a ball from it. The ball is white. Then the probability that the ball came from the first urn is equal to:

A) $\frac{7}{12}$ B) $\frac{5}{12}$

C) $\frac{7}{34}$ D) $\frac{5}{34}$

27. Given two distribution functions F(x) and G(x). Then which of the following is not a distribution function

A) $\frac{F(x)+G(x)}{2}$ B) $\frac{[F(x)]^2+[G(x)]^2}{2}$

C) $1-\frac{[1-F(x)]^2+[1-G(x)]^2}{2}$ D) $1-\frac{[F(x)]^2+[G(x)]^2}{2}$

28. Identify the distribution of a random variable X whose moment generating function is $M(t) = e^{\lambda(e^t-1)}$. The distribution of X is

A) Binomial distribution B) Hyper geometric distribution

C) Poisson distribution D) Geometric distribution

29. Name the distribution whose characteristic function is given by $\phi(t) = e^{-|t|}$,

A) Laplace distribution B) Cauchy distribution

C) Normal distribution D) Logistic distribution

30. Let X be a standard normal variate and $\{X_n\}$ be a sequence of random variables such that $X_{2m-1} = X$ and $X_{2m} = -X$ for m = 1, 2, … Now consider the statements:

i) $X_n \xrightarrow{L} X$ ii) $X_n \xrightarrow{P} X$

Now state which of the following is correct?

A) Only (i) is true B) Only (ii) is true

C) Both (i) and (ii) are true D) Neither (i) nor (ii) is true

31. Given {An} as a sequence of events such that $\sum_{n=1}^{\infty} P(An) < \infty$. Now if we define $A = \limsup_{n\to\infty} An$, then

A) P(A) = 0 B) P(A) = 1 C) P(A) < 1 D) P(A) > 0

32. The probability density function of a bivariate random vector (X, Y) is given by

$$f(x,y) = \begin{cases} 2, 0 < x < y < 1 \\ 0, \text{otherwise.} \end{cases}$$

Then the marginal pdf of Y is given by

A) $f_2(y) = \begin{cases} 2-2y, 0 < y < 1 \\ 0, \text{otherwise.} \end{cases}$ B) $f_2(y) = \begin{cases} 2y, 0 < y < 1 \\ 0, \text{otherwise.} \end{cases}$

C) $f_2(y) = \begin{cases} 1, 0 < y < 1 \\ 0, \text{otherwise.} \end{cases}$ D) $f_2(y) = \begin{cases} 3y^2, 0 < y < 1 \\ 0, \text{otherwise.} \end{cases}$

33. Let X be a non negative random variable with mean and standard deviation σ. Now for c > 0 consider the statements

(i) $P\{|X - \ | \geq c\} \leq \frac{E[X - \ ]^r}{c^r}, r > 0$ (ii) $P\{|X - \ | < c\sigma\} \geq 1 - \frac{1}{c^2}$

Then state which of the following is correct?

A) (i) only is true B) (ii) only is true

C) (i) and (ii) are both false D) Both (i) and (ii) are true

34. The random variable X has Poisson distribution with pmf $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$, $x = 0, 1, 2, \ldots$. If $f(2) = 2f(0)$ is the case with $f(x)$, then the value of λ is equal to:

A) 1 B) 2 C) 0.5 D) 4

35. For a non-negative random variable X with continuous distribution it is observed that $P(X > x+y / x > y) = P(X > x)$ for $x, y \in R^+$. Then the distribution of x is

A) Uniform B) Exponential

C) Gamma D) Weibull

36. Suppose X is distributed as standard exponential distribution. Let Q^+ be the set of all positive rational numbers. Then $P(X \in Q^+)$ is equal to

A) 1 B) $\frac{1}{2}$

C) 0 D) $\sum_{x \in Q^+} e^{-x}$

37. Suppose the mean deviation about the mean of a normal distribution is $\sqrt{2}$, then the standard deviation of the distribution is equal to:

A) $\sqrt{\frac{2}{\pi}}$ B) $\sqrt{\frac{\pi}{2}}$

C) $\sqrt{\pi}$ D) $\frac{1}{\sqrt{2\pi}}$

38. If X and Y are two independent random variables with a common distribution which is negatively skewed, then the distribution of X – Y is

A) Negatively skewed B) Positively skewed
C) Symmetric D) Uniform

39. Given X_1, X_2, X_3 are iid random variables with support R and it is known that $l_1 X_1 + l_2 X_2 + l_3 X_3$ is normally distributed for any arbitrary scalars l_1, l_2 and l_3. Then the common distribution of X_1, X_2 and X_3 is

A) Laplace B) Normal
C) Exponential D) Cauchy

40. The pdf of standard normal variate X is denoted by f(x) and its distribution function is denoted by F(x). Then the distribution of Y = F(X) is

A) Uniform over (0,1) B) Exponential
C) Cauchy D) Gamma

41. Let X be a random variable with an absolutely continuous unimodal distribution. Consider the statements

(i) Mean always exists (ii) Variance always exists
(iii) Median always exists (iv) Mode always exists.

Now state which of the above statements is/ are correct?

A) (i) only B) (i) and (ii) only
C) (iii) and (iv) only D) (ii) only

42. Given $X_{2:3}$ is the second order statistic of a random sample of size 3 drawn from a distribution with cdf F(x). Then the cdf of $X_{2:3}$ is given by

A) $F^2(x)$ B) $3 F^3(x) - 2 F^2(x)$
C) $F(x)\,(1-F(x))$ D) $3F^2(x) - 2F^3(x)$

43. Given $X_{3:5}$ is the median of a random sample of size 5 drawn from a population with cdf F(x) and pdf f(x). Then the pdf of $X_{3:5}$ is

A) $15 F^3(x)\,[1-F(x)]^2 f(x)$ B) $15[F(x)]^2\,[1-F(x)]^3$
C) $30[F(x)]^2[1-F(x)]^2 f(x)$ D) $30F(x)[1-F(x)]\,f(x)$

44. Suppose $X_{r:n}$ is the r^{th} order statistic of a random sample of size n drawn from the standard exponential distribution. Then the mean $_{r:n}$ and variance $\sigma^2_{r:n}$ of $X_{r:n}$ are

A) $_{r:n} = \sum_{i=r}^{n} \frac{1}{(n-i+1)}, \sigma^2_{r:n} = \sum_{i=r}^{n} \frac{1}{(n-i+1)^2}$

B) $_{r:n} = \sum_{i=1}^{r} \frac{1}{(n-i+1)}, \sigma^2_{r:n} = \sum_{i=1}^{r} \frac{1}{(n-i+1)^2}$

C) $_{r:n} = \frac{1}{n-r+1}, \sigma^2_{r:n} = \frac{1}{(n-r+1)^2}$

D) $_{r:n} = \frac{r}{n}, \sigma^2_{r:n} = \frac{r^2}{n(n-1)}$

45. If $X_{r:n}$ is the r^{th} order statistic of a random sample of size n drawn from the uniform distribution over(0,1), then the distribution of $X_{r:n}$ is:

A) Beta distribution of first kind
B) Beta distribution of second kind
C) Gamma distribution
D) t – distribution

46. Let X_1 and X_2 be two independent observations drawn from the exponential distribution with pdf $f(x) = \frac{1}{2} e^{-x/2}, x > 0$. Then the distribution of $X_1 + X_2$ is

A) Exponential
B) Chi-square with 2 degrees of freedom
C) Chi-square with 4 degrees of freedom
D) Generalized exponential

47. Let X_1 and X_2 be two independent observations drawn from the exponential distribution with pdf $f(x) = \frac{1}{2} e^{-x/2}, x > 0$. Then the distribution of $Y = X_1/X_2$ is:

A) F with (1,1) degrees of freedom
B) F with (2,2) degrees freedom
C) Student's t-distribution with 2 degrees of freedom
D) Student's t-distribution with 4 degrees of freedom

48. Given the random variable X follows student's t-distribution with one degree of freedom and the following statements:

(i) Mean of the distribution of X is zero
(ii) Median of the distribution of X is zero

Now state which of the following is correct?

A) (i) only
B) (ii) only
C) Both (i) and (ii)
D) Neither (i) nor (ii) is correct

49. Suppose (x_1,x_2,x_3) represent the outcomes of a biased coin in three successive tosses with x_i represents the outcome 1 for head turning up with probability p and 0 for tail turning up with probability $q = 1 – p$ in the i^{th} toss for $i = 1,2,3$. Let $T = x_1+x_2+x_3$ be a statistic. Then the equivalent classes E_0, E_1, E_2, E_3 determined by T as a partition in the sample space are given by:

A) $E_0 = \{(0, 0, 0)\}$, $E_1 = \{(1, 0, 0),(0, 1, 0),(0, 0, 1)\}$, $E_2 = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$, $E_3 = \{(1, 1, 1)\}$
B) $E_0 = \{(0, 0, 0)\}$, $E_1 = \{(1, 0, 0),(1, 1, 0)\}$, $E_2 = \{(0, 1, 1),(1, 0, 1)\}$, $E_3 = \{(0, 0, 1),(0, 1, 0),(1, 1, 1)\}$
C) $E_0 = \{(0, 0, 0)\}$, $E_1 = \{(1, 0, 0),(0, 1, 0),(0, 0, 1)\}$, $E_2 = \{(1, 1, 0),(1, 0, 1)\}$, $E_3 = \{(0, 1, 1),(1, 1, 1)\}$
D) $E_0 = \{(0, 0, 0),(1, 1, 1)\}$, $E_1 = \{(1, 0, 0),(1, 0, 1)$, $E_2 = \{(0, 1, 0),(0, 0, 1)\}$, $E_3 = \{(1, 1, 0),(0, 1, 1)\}$

50. Let $\overline{X}$ be the mean and m be the median of a random sample of size n drawn from the distribution with pdf $f(x,\theta) = \frac{1}{\pi}\frac{1}{1+(x-\theta)^2}, -\infty < x < \infty, -\infty < \theta < \infty$. Consider the following statements:

(i) $\overline{X}$ is unbiased for θ (ii) $\overline{X}$ is a consistent estimator of θ
(iii) m is a consistent estimator of θ .

Now state which of the above statements are correct?

A) (i) only B) (ii) only
C) Both (i) and (ii) D) (iii) only

51. Consider $X_1, X_2, \ldots, X_n$ as a random sample arising from $N(\theta,\theta^2)$, $\theta > 0$. Then a sufficient statistic for θ is

A) $\sum_{i=1}^{n} X_i$ B) $\sum_{i=1}^{n} X_i^2$

C) $\left(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i^2\right)$ D) $\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$

52. IF T is the UMVUE of θ and T' is any other unbiased estimator of θ with efficiency e, then the correlation coefficient between T and T' is equal to

A) e B) $\sqrt{e}$
C) e^2 D) $1/e$

53. If $X_1, X_2, \ldots, X_n$ is a random sample of size n drawn from $N(\mu,\sigma^2)$, then the UMVUE of σ^2 is

A) $\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$ B) $\frac{1}{n}\sum_{i=1}^{n}\left|X_i - \overline{X}\right|$

C) $\frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$ D) $\frac{1}{n}T^2$ where $T = \frac{1}{n}\sum_{i=1}^{n}\left|X_i - \overline{X}\right|$

54. Let $\overline{X}$, M, m and R be the mean, median, mode and range of a random sample of size n drawn from the distribution with pdf $f(x,\theta) = \frac{1}{2}e^{-|x-\theta|}, -\infty < x < \infty, -\infty < \theta < \infty$. Then the MLE of θ is

A) $\overline{X}$ B) M
C) m D) R

55. Let $X_{1:n}, X_{2:n}, \ldots, X_{n:n}$ be the order statistics of a random sample of size n drawn from the uniform distribution over $\left(\theta-\frac{1}{2}, \theta+\frac{1}{2}\right)$. Consider the statements given below on the estimator $T = \frac{X_{1:n}+X_{n:n}}{2}$:

(i) T is unbiased for θ (ii) T is MLE of θ (iii) T is UMVUE of θ

Now state which of the above statements is/are true?

A) (i) only B) (ii) only

C) both (i) and (ii) D) (iii) only

56. Given a random sample $X_1, X_2, \ldots, X_n$ of size n from a distribution with pdf $f(x,\theta) = \frac{1}{2\theta} e^{-\frac{|x|}{\theta}}, -\infty < x < \infty, \theta > 0$. Then the method of moments estimator of θ is

A) $\overline{X} = \sum_{i=1}^{n} \frac{X_i}{n}$ B) M, the sample median

C) $S = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(X_i-\overline{X}\right)^2}$ D) $\sqrt{\frac{1}{2n}\sum_{i=1}^{n} X_i^2}$

57. Suppose $\overline{X}$ and S are the mean and standard deviation of a random sample of size n drawn from N (μ,σ^2). Let $t_{.025}$ be the $(0.975)^{th}$ quantile of student's t– distribution with n-1 degrees of freedom. Then a 95% confidence interval for is

A) $\overline{X} \pm 1.96\frac{S}{\sqrt{n}}$ B) $\overline{X} \pm 1.96\frac{S}{\sqrt{n-1}}$

C) $\overline{X} \pm t_{.025}\frac{S}{\sqrt{n-1}}$ D) $\overline{X} \pm t_{.025}\frac{S}{\sqrt{n}}$

58. Given that ϕ is a size α test of $H_0:\theta \in P_0$ against $H_1:\theta \in P_1$ and unbiased. Then which of the following is true on the power function $\beta_\phi(\theta)$?

A) $\beta_\phi(\theta) \le \alpha$, for $\theta \in P_0$ and $\beta_\phi(\theta) \le \alpha$, for $\theta \in P_1$

B) $\beta_\phi(\theta) \ge \alpha$, for $\theta \in P_0$ and $\beta_\phi(\theta) \ge \alpha$, for $\theta \in P_1$

C) $\beta_\phi(\theta) \ge \alpha$, for $\theta \in P_0$ and $\beta_\phi(\theta) \le \alpha$, for $\theta \in P_1$

D) $\beta_\phi(\theta) \le \alpha$, for $\theta \in P_0$ and $\beta_\phi(\theta) \ge \alpha$, for $\theta \in P_1$

59. Given the distributions

(i) uniform over (0,θ) (ii) Cauchy with pdf $f(x,\theta) = \frac{1}{\pi}\frac{1}{1+(x-\theta)^2}, -\infty < x < \infty$

(iii) Normal distribution with mean θ and variance one.

Then Monotone Likelihood ratio property exists for the distribution in

A) (iii) only B) (i) only

C) (ii) only D) (i) and (iii) only

60. Given $Y = a_1 X_1 + a_2 X_2 + a_0$ is a multiple regression equation fitted based on the observations (Y_i, X_{1i}, X_{2i}) $i = 1, 2, \ldots, 21$ of a data. Further it is given that $\sum_{i=1}^{21}\left(Y_i - \overline{Y}\right)^2 = 540$ and $\sum_{i=1}^{21}\left(Y_i - \hat{Y}_i\right)^2 = 324$ where $\hat{Y}_i$ is the estimated value of Y_i by the regression. Now consider the following statements:

(i) The degrees of freedom for the statistic to be used for testing H_0: $a_1 = a_2 = 0$ is (2,18)

(ii) The value of F-statistic to be used for testing H_0 is 6

(iii) The multiple correlation coefficient of Y on X_1 and X_2 is $\sqrt{\frac{2}{5}}$.

Now state which of the following is correct?

A) (i), (ii) and (iii) B) (i) and (ii) only
C) (i) and (iii) only D) (ii) and (iii) only

61. In a tri-variate population with variables X_1, X_2, X_3 given that the simple correlation between any two variables is equal to ½. Now consider the following statements:

(i) $1 - R^2_{1.23} = \frac{2}{3}$, (ii) $r_{12.3} = \frac{1}{3}$, (iii) $R^2_{2.13} = \frac{2}{3}$.

Which of the above statements are correct?

A) (i), (ii) and (iii) B) (i) and (ii)
C) (i) and (iii) D) (ii) and (iii)

62. Let (X_i, Y_i), $i = 1, 2, \ldots, n$ be observations recorded on units where X_i is the pre-treatment score and Y_i is the post-treatment score. Let $\overline{X}$ and S_1 be the mean and standard deviation of the X_i-values, $\overline{Y}$ and S_2 be the mean and standard deviation of Y_i-values. If $\overline{d}$ and S_d are the mean and standard deviation of $d_i = Y_i - X_i$, $i = 1, 2, \ldots, n$. Then to test H_0: The treatment has no effect on the units, which of the following test statistic is used?

A) $\dfrac{\overline{X} - \overline{Y}}{\sqrt{\dfrac{S_1^2 + S_2^2}{n-1}}}$ B) $\dfrac{\overline{X} - \overline{Y}}{\sqrt{\left[S_d^2 / (n-1)\right]}}$

C) $\dfrac{\overline{d}}{\left[S_d / \sqrt{n-1}\right]}$ D) $\dfrac{\overline{d}}{\sqrt{\dfrac{S_1^2 + S_2^2}{n-1}}}$

63. If two sample runs test is used to test Ho: the two samples which are given below arises from the same population

Sample 1 : 1.3, 1.4, 1.4, 1.5, 1.7, 1.9, 1.9

Sample 2 : 1.6, 1.8, 2.0, 2.1, 2.1, 2.2, 2.3

then the number of runs to be used as a test statistic is:

A) 6 B) 14 C) 5 D) 8

64. For a frequency distribution, normal distribution was fitted. The observed and expected frequencies in the various classes are given below

Class	1	2	3	4	5	6	7
Observed Frequency	5	8	15	20	19	16	7
Expected Frequency	4	12	18	22	18	12	4

Then to test for the goodness of fit using chi-square statistic what is the degrees of freedom of the statistic?

A) 1 B) 2 C) 4 D) 6

65. The probability density function of the random variable X is given by

$$f(x) = \begin{cases} 3x^2, 0 < x < 1 \\ 0, \text{otherwise} \end{cases}$$

If 3 random numbers selected from the interval (0,1) are 0.125, 0.008, 0.216, then which of the following shall be considered as a random sample of size three from the above distribution?

A) 0.125, 0.008, 0.216 B) 0.250, 0.016, 0.432
C) 0.375, 0.024, 0.648 D) 0.50, 0.20, 0.60

66. In order to draw a simple random sample of size 5 without replacement from a population of 30 units, the following two digit random numbers: 68, 44, 08, 97, 83, 12, 72, 56 are given. Then which of the following set of units shall be considered as the required sample?

A) 08, 14, 08, 07, 23 B) 08, 14, 07, 23, 12
C) 08, 14, 07, 23, 26 D) 08, 14, 23, 12, 26

67. If srswr is used to draw a sample of three units from a population of N units, then the probability that the sample contains different units is

A) $\frac{1}{N^3}$ B) $\frac{1}{NC_3}$

C) $\frac{3}{N^3}$ D) $\frac{(N-1)(N-2)}{N^2}$

68. If y_i, x_i represent the measurements made on the variates X and Y respectively on the i^{th} unit of a simple random sample of size n for i = 1, 2, …, n and $R = \overline{Y} / \overline{X}$, then for large n, the variance of $\hat{R} = \overline{y} / \overline{x}$ is:

A) $\sum_{i=1}^{N} \frac{(y_i - Rx_i)^2}{N-1}$ B) $\frac{N-n}{N(N-1)} \sum_{i=1}^{N} \frac{(y_i - Rx_i)^2}{n\overline{X}^2}$

C) $\sum_{i=1}^{N} \frac{(y_i - Rx_i)^2}{N\overline{X}^2}$ D) $\sum_{i=1}^{N} \frac{(y_i - Rx_i)^2}{(N-1)\overline{X}^2}$

69. In a survey, the aim is to estimate the population mean $\overline{Y}$ such that the probability of large deviation of the sample mean $\overline{y}$ with $\overline{Y}$ exceeds d is equal to α. Assume $\overline{y}$ is normally distributed and t as the abscissa of the normal curve that cuts off an area α at the tails. Then the sample size required for srs (for a population with mean square error S^2) is approximately equal to:

A) $\dfrac{[(tS)/d]^2}{1+N^{-1}[(tS)/d]^2}$ B) $\dfrac{1-\alpha}{\alpha}[(tS)/d]^2$

C) $\dfrac{\alpha}{1-\alpha}[(tS)/d]^2$ D) $\alpha[(tS)/d]^2$

70. Which of the following set of units shall be considered as a systematic sample of size 6 drawn from a population of 43 units?

A) 10, 16, 22, 28, 34, 40 B) 5, 11, 17, 23, 29, 35
C) 15, 22, 29, 36, 43, 7 D) 6, 13, 20, 27, 34, 41, 7

71. If a population consists of a linear trend, then consider the following statements with regard to the removal of the effect of the trend:

(i) Stratified sampling is more effective than the systematic sampling
(ii) Stratified sampling is more effective than simple random sampling
(iii) Systematic sampling is more effective than the simple random sampling

Now which of the above statements are true?

A) (i) and (ii) only B) (i) and (iii) only
C) (ii) and (iii) only D) (i), (ii) and (iii)

72. If Y_1 and Y_2 are independent variates with common variance σ^2 and means given by $E(Y_1) = \theta_1 + \theta_2$, $E(Y_2) = \theta_2$, then consider the following statements

(i) Every linear parametric function $\ell_1\theta_1 + \ell_2\theta_2$ is estimable
(ii) BLUE of θ_1 is $Y_1 - Y_2$
(iii) BLUE of θ_2 is Y_2

Now state which of the above statements is/are true?

A) (i) only B) (ii) and (iii) only
C) (i) and (ii) only D) (i), (ii) and (iii)

73. Consider the Gauss-Markov setup $(Y, A\theta, \sigma^2 I_3)$, where $Y = (Y_1, Y_2, Y_3)'$ $\theta = (\theta_1, \theta_2, \theta_3)'$, $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ and I_3 is a unit matrix of order 3. Then among $b_1'\theta = \theta_1 - \theta_2 + \theta_3$, $b_2'\theta = \theta_1 + \theta_2 + \theta_3$, which of the following is true?

A) $b_1'\theta$ is estimable but $b_2'\theta$ is not estimable
B) $b_1'\theta$ is not estimable but $b_2'\theta$ is estimable
C) Both $b_1'\theta$ and $b_2'\theta$ are not estimable
D) Both $b_1'\theta$ and $b_2'\theta$ are estimable

74. Consider the following 4 x 4 Latin squares:

	B	D	A	C		b	d	a	c		1	2	3	4
L_1:	A	C	B	D	L_2:	d	b	c	a	L_3:	2	3	4	1
	D	B	C	A		c	a	d	b		3	4	1	2
	C	A	D	B		a	c	b	d		4	1	2	3

and the statements:
(i) L_1 and L_2 are orthogonal (ii) L_1 and L_3 are orthogonal (iii) L_2 and L_3 are orthogonal. Now state which of the above statements is/are true?

A) (i) only
B) (ii) only
C) (iii) only
D) (ii) and (iii) only

75. Consider the following linear functions of observations
$L_1 = Y_1 + Y_2 - Y_3 - Y_4$, $L_2 = Y_1 + Y_2 + Y_3 + Y_4$, $L_3 = Y_1 - Y_2 - Y_3 + Y_4$
$L_4 = -Y_1 + Y_2 - Y_3 + Y_4$ and the statements
(i) L_1, L_3 and L_4 are linear contrasts.
(ii) L_1 and L_3 are orthogonal contrasts but L_3 and L_4 are not so.
(iii) L_1, L_3 and L_4 are mutually orthogonal contrasts.
Now state which of the above statements is/are correct?

A) (i) only
B) (ii) only
C) (i) and (ii) only
D) (i) and (iii) only

76. An experimenter has planned to have 10 treatments, 6 replications and 4 plots in a block of BIBD. Then for designing it what should be the minimum number of blocks that he may start with in the search of an admissible design?
A) 12 B) 10 C) 20 D) 11

77. The following is the layout and yield observed in a 4 x 4 LSD with a single observation missing

B 7	D 4	A 5	C 8
A 6	C 9	B 6	D 3
D 4	B 7	C 7	A 5
C 10	A 5	D –	B 8

The estimated value of the missing cell is equal to:
A) $\frac{11}{3}$
B) $\frac{10}{3}$
C) $\frac{23}{3}$
D) 6

78. In a 2^3–experiment with factors A, B and C, two interactions are completely confounded into the blocks and the principal block is obtained as {(1), abc}. Then the interactions confounded are

A)	A, BC, ABC	B)	B, BC, ABC
C)	C, AB, ABC	D)	AB, AC, BC

79. If the principal block of confounding in a 2^3-experiment with factors A, B and C is {(1), abc} and the same confounding system is repeated four times, then the degrees of freedom for error is

A) 10 B) 12 C) 15 D) 16

80. In a 3^3-experiment, the interactions ABC, ABC^2, AB^2C and AB^2C^2 are partially confounded in different replications. Then in the analysis, the degrees of freedom for the error is equal to:

A) 37 B) 71 C) 70 D) 78

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